NATURAL PHILOSOPHY

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PREFACE

The beginning of the twentieth century is marked by a sudden rise of interest in philosophy. This is especially manifest in the vast growth of philosophic literature. The present movement, it is noteworthy, is by no means a revival proceeding from the academic philosophy traditionally represented at the universities, but has rather the original character of *natural philosophy*. It owes its origin to the fact that after the specialization of the last half century, the synthetic factors of science are again vigorously asserting themselves. The need finally to consider all the numerous separate sciences from a general point of view and to find the connection between one's own activity and the work of mankind in its totality, must be regarded as the most prolific source of the present philosophic movement, just as it was the source of the natural philosophic endeavors a hundred years ago.

But while that old natural philosophy soon ended in a boundless sea of speculation, the present movement gives promise of permanent results, because it is built upon an extremely broad basis of experience. The laws of energy in the inorganic world and the laws of evolution in the organic world fur-

nish mental instruments for a conceptual elaboration of the material provided by science, instruments capable not only of unifying present knowledge, but also of evoking the knowledge of the future. If it is not permissible to regard this unification as exhaustive and sufficient for all time, yet there is still so much left for us to do in working over the material we have on hand from the general points of view just mentioned, that the need for systematizing must be satisfied before we can turn our gaze upon things more remote.

The present work is meant to serve as the first aid and guide in the acquisition of these comprehensive notions of the external world and the inner life. It is not meant to develop or uphold a "system of philosophy." Through long experience as a teacher the writer has learned that those are the best pupils who soon go their own way. However, it *is* meant to uphold a certain method, that is, the scientific (or, if you will, the *natural* scientific), which takes its problems, and endeavors to solve its problems, from experience and for experience. If, as a result, several points of view arise that differ from those of the present day, and consequently demand a different attitude toward important matters in the immediate future, this very fact affords proof that our present natural philosophy does not lead away from life, but aims to form a part of our life, and has a right to.

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INTRODUCTION

Natural science and natural philosophy are not two provinces mutually exclusive of each other. They belong together. They are like two roads leading to the same goal. This goal is the domination of nature by man, which the various natural sciences reach by collecting all the individual actual relations between the natural phenomena, placing them in juxtaposition, and seeking to discover their interdependence, upon the basis of which one phenomenon may be foretold from another with more or less certainty. Natural philosophy accompanies these specialized labors and generalizations with similar labors and generalizations, only of a more universal nature. For instance, while the science of electricity, as a branch of physics, deals with the relation of electrical phenomena to one another and to phenomena in other branches of physics, natural philosophy is not only concerned with the question of the mutual connection of *all* physical relations, but also endeavors to include in the sphere of its study chemical, biological, astronomical, in short, all the known phenomena. In other words, *natural philosophy is the most general branch of natural science.*

Here two questions are usually asked. First, how can we define the boundary line between natural philosophy and the special sciences, since, obviously, sharp lines of demarcation are out of the question? Secondly, how can we investigate and teach natural philosophy, when it is impossible for any one person to master all the sciences completely, and so obtain a bird's-eye view of the general relations between all the branches of knowledge? To the beginner especially, who must first learn the various sciences, it seems quite hopeless to devote himself to a study that presupposes a command of them.

Since a discussion of the two questions will afford an excellent preliminary survey of the work in hand, it will be well to consider them in detail. In the first place, *the lack of complete and precise boundary lines is a general characteristic of all natural things,* and science is a natural thing. If, for instance, we try to differentiate sharply between physics and chemistry, we are met with the same difficulty. So also in biology if we try to settle beyond the shadow of a doubt the line of separation between the animal and the vegetable kingdoms.

If, despite this well-known impossibility, we consider the division of natural things into classes and orders as by no means useless and do not discard it, but regard it as an important scientific work, this is practical proof that such classification preserves its essential usefulness, even if it does not attain ideal definiteness. For, this imperfection

notwithstanding, classification reaches its end, which is a comprehensive view, and thus a mastery, of the manifoldness of phenomena. For example, with the overwhelming majority of organic beings there is no doubt whether they are animals or plants. Similarly, most phenomena of inorganic nature can readily be designated as physical or chemical. For all such cases, therefore, the existing classification is good and useful. The few cases presenting difficulty may very well be considered by themselves wherever they occur, and we need merely take cognizance of them here. It follows from this, to be sure, that classification will be all the *better fitted to its purpose the less frequently* such doubtful cases arise, and that we have an interest in repeatedly testing existing classifications with a view to finding out if they cannot be supplanted by more suitable ones.

In these matters it is much the same as when we look upon the waves on the surface of a large body of water. Our first glance tells us that a number of waves are rolling there; and from a point giving us a sufficiently wide outlook, we can count them and gauge their width. But where is the line of division between one wave and the next? We undoubtedly see one wave following another, yet it is impossible for us to indicate precisely the end of one and the beginning of the next. Are we then to deduce that it is superfluous or unfeasible to designate the waves as different? By no means. On

the contrary, in strictly scientific work we will endeavor to find some suitable definition of the boundary line between two consecutive waves. It may then be called an arbitrary line, and in a degree arbitrary it will certainly be. But to the investigator this does not matter. What concerns him is, if, with the help of this definition, wave lengths can be unequivocally determined, and if this is possible, he will use the definition as suitable to the purposes of science, without dismissing from his mind the idea that possibly some other definition may provide an even easier or sharper determination. Such an one he would instantly prefer to the old one.

Thus we see that these questions of classification are not questions of the so-called "essence" of the thing, *but pertain merely to purely practical arrangements for an easier and more successful mastery of scientific problems.* This is an extremely important point of view, much more far-reaching than is apparent here at its first application.

As to the second objection, I will admit its validity. But here, too, we have a phenomenon appearing in all branches and forms of science. Therefore we must familiarize ourselves with it in advance. Science was created by man for man's purposes, and, consequently, like all human achievements, possesses the indestructible quality of imperfection. But the mere fact that a successful working science exists, with the help of which human life

has been fundamentally modified, signifies that *the quality of incompleteness in human learning is no hindrance to its efficiency.* For what science has once worked out always contains a portion of truth, hence a portion of efficiency. The old corpuscular theory of light, which now seems so childishly incomplete to us, was adequate, none the less, for satisfactorily explaining the phenomena of reflection and refraction, and the finest telescopes have been built with its help. This is due to the *true elements* in it, which taught us correctly to calculate the direction of rays of light in reflection and refraction. The rest was merely an arbitrary accessory which had to fall when new, contradictory facts were discovered. These facts could not have been taken into consideration when the theory was propounded, because they were not yet known. But when the corpuscular theory of light was replaced by the theory of waves of an elastic ether, geometric optics at first remained quite unchanged, because the theory of straight lines of rays could be deduced from the new views also, though not so easily and smoothly. And geometric optics was then concerned with nothing but these straight lines, in no wise with the question of their propagation. It did not become clear until recently that this conception of straight lines of rays is incomplete, though, it is true, it made a first approach toward the presentation of actual phenomena. It fails when it comes to characterize the behavior of a pencil of rays of

large aperture. The old idea of a straight line of rays was to be replaced by a more complex concept with more varied characteristics, namely, the wave-surface. The greater variety of this concept renders possible the presentation of the greater variety of the optical phenomena just mentioned. And from it proceed the very considerable advances that have been made, since the new theory was propounded, in optical instruments, especially the microscope and the photographic objective, for the purposes of which pencils of rays of large aperture are required. The astronomic objective with its small angle of aperture has not undergone particularly important improvements.

Experience in every province of science is the same as in this. Science is not like a chain which snaps when only a single link proves to be weak. It is like a tree, or, better still, like a forest, in which all sorts of changes or ravages go on without causing the whole to pass out of existence or cease to be active. The relations between the various phenomena, once they become known, continue to exist as indestructible components of all future science. It may come to pass, in fact, does come to pass very frequently, that the form in which those relations were first expressed prove to be imperfect, and that the relations cannot be maintained quite generally. It turns out that they are subjected to other influences which change them because they had been unknown, and which could not have been taken

into consideration at the discovery and first formulation of these relations. But no matter what changes science may undergo, a certain residue of that first knowledge will remain and never be lost. In this sense, a truth that science has once gained has life eternal, that is, it will exist as long as human science exists.

Applying this general notion to our case, we have the following. How far and how generally at any given time the relations of the various phenomena are summed up in fixed forms, that is, in natural laws, will depend upon the stage attained by each of the special sciences. But since science has been in existence it has yielded a certain number of such general laws, and these, though they have been filed down a good deal in form and expression, and have undergone many corrections as to the limits of their application, nevertheless have preserved their essence, since they began their existence in the brains of human investigators. The net of the relations of phenomena grows ever wider and more diversified, but its chief features persist.

The same is true of an individual. No matter how limited the circle of his knowledge, *it is a part of the great net, and therefore possesses the quality by virtue of which the other parts readily join it as soon as they reach the consciousness and knowledge of the individual.* The man who thus enters the realm of science acquires advantages which may be compared to those of a telephone in his residence.

If he wishes to, he may be connected with everybody else, though he will make extremely limited use of his privilege, since he will try to reach only those with whom he has personal relations. But once such relations have been established, the possibility of telephone communication is simultaneously and automatically established. Similarly, every bit of knowledge that the individual appropriates will prove to be a regular part of the central organization, the entire extent of which he can never cover, though each individual part has been made accessible to him, provided he wants to take cognizance of it.

The mere beginner in learning, therefore, when receiving the most elementary instruction in school, or from his parents, or even from his personal experiences in his surroundings, is grasping one or more threads of the mighty net, and can grope his way farther along it in order to draw an increasing area of it into his life and the field of his activity. *And this net has the valuable, even precious quality of being the same that joins the greatest and most comprehensive intellects in mankind to one another.* The truths a man has once grasped he need never learn afresh so far as their *actual content* is concerned, though not infrequently—especially in newer sciences—he may have to see the *form* of their presentation and generalization change. For this reason it is of such especial importance for each individual from the first to perceive these unalterable facts and realize that they are unalterable and learn

to distinguish them from the alterable forms of their presentation. It is in this very regard that the incompleteness of human knowledge is most clearly revealed. Time and again in the history of science form has been taken for content, and necessary changes of form—a merely practical question—have been confused with revolutionary modifications of the content.

Thus, each presentation of a science has its natural philosophic portion. In text-books, whether elementary or advanced, the chapter on natural philosophy is found usually at the beginning of the book, sometimes at the end, in the form of a "general introduction," or "general summary." In the special works in which the latest advances of science are made known by the investigators, the natural philosophic portions are usually to be found in the form of theses, of principles, which are not discussed, often not even explicitly stated, but upon the acceptance of which depend all the special conclusions that are drawn, in the case in hand, from the new facts or thoughts imparted. Whether at the beginning or at the end of the book, these most general principles do not quite occupy the place that befits them. If at the introduction of the text-book, they are practically devoid of content, since the facts they are meant to summarize are yet to be unfolded in the course of the presentation. If at the end, they come too late, since they have already been applied in numerous instances, though without refer-

ence to their general nature. The best method is—and a good teacher always employs this method, whether in the spoken or the written word—to let the generalizations come whenever the individual facts imparted require and justify them.

Thus, all instruction in natural sciences is necessarily interspersed with natural philosophy, good or bad, according to the clearheadedness of the teacher. If we wish to obtain a perfect survey of a complex structure, as, for instance, the confusion of streets in a large city, we had better not try to know each street, but study a general plan, from which we learn the comparative situation of the streets. So it is well for us in studying a special science to look at our general plan, if for no other reason than to keep from losing our way when it may chance to lead through a quarter hitherto unknown. This is the purpose of the present work.

PART I

GENERAL THEORY OF KNOWLEDGE

1. **The Formation of Concepts.** To the human mind, as it slowly awakens in every child, the world at first seems a chaos consisting of mere individual experiences. The only connection between them is that they follow each other consecutively. Of these experiences, all of which at first are different from one another, certain parts come to be distinguished by the fact that they are repeated more frequently, and therefore receive a special character, that of *being familiar*. The familiarity is due to our *recalling* a former similar experience; in other words, to our feeling that there is a relation between the present experience and certain former experiences. The cause of this phenomenon, which is at the basis of all mental life, is a quality common to all living things, and manifesting itself in all their functions, while appearing but rarely or accidentally in inorganic nature. It is the quality by virtue of which *the oftener any process has taken place in a living organism the more easily it is repeated.* Here is not yet the place to show how almost all the characteristic qualities of living

beings, from the preservation of the species to the highest intellectual accomplishments, are conditioned by this special peculiarity. Suffice it to say that because of this quality all those processes which are repeated frequently in any given living organism, assume spontaneously, that is, from physiologic reasons, a character distinguishing them essentially from those which appear only in isolated instances, or sporadically.

If a living being is equipped with consciousness and thought, like man, then the conscious recollections of such uniform experiences form the enduring or permanent part in the sum-total of his experiences. Each time a complex event, like the change of seasons, for example, which we know from experience repeats itself—each time a part of such an event reaches our consciousness, we are prepared also for the other parts that experience teaches are connected with it. This makes it possible for us to foresee future events. What significance the foreseeing of future events has for the preservation and the development of the individual as well as the species can only be indicated here. To give one instance, it is our ability to foretell the coming of winter with the impossibility of obtaining food directly during the winter that causes us to refrain from at once using up all the food we have and to preserve it for the day of need. The ability to foretell, therefore, becomes the foundation of the whole structure of economic life.

2. Science. The prophecy of future events based upon the knowledge of the details of recurring events is called *science* in its most general sense. Here, as in most cases in which language became fixed long before men had a clear knowledge of the things designated, the name of the thing is easily associated with false ideas arising either from errors that had been overcome or from other, still more accidental, causes. Thus, the mere knowledge of *past* events is also called science without any thought of its use for prophesying future events. Yet a moment's reflection teaches that mere knowledge of the past which is not meant to, or cannot, serve as a basis for shaping the future is utterly aimless knowledge, and must take its place with other aimless activities called *play*. There are all sorts of plays requiring great acumen and patient application, as for example the game of chess; and no one has the right to prevent any individual from pursuing such games. But the player for his part must not demand special regard for his activity. By using his energies for his personal pleasure and not for a social purpose, that is, for a general human purpose, he loses every claim to the social encouragement of his activity, and must be content if only his individual rights are respected; and that, too, only so long as the social interests do not suffer by it.

3. The Aim of Science. These views are deliberately opposed to a very widespread idea that science should be cultivated "for its own sake," and not for

the sake of the benefits it actually brings or may be made to bring. We reply that there is nothing at all which is done merely " for its own sake." Everything, without exception, is done for human purposes. These purposes range from momentary personal satisfaction to the most comprehensive social services involving disregard of one's own person. But in all our actions we never get beyond the sphere of the human. If, therefore, the phrase " for its own sake " means anything, it means that science should be followed for the sake of the immediate pleasure it affords, that is to say, as *play* (as we have just characterized it), and in the " for-its-own-sake " demand there is hidden a misunderstood idealism, which, on closer inspection, resolves itself into its very opposite, the degradation of science.

The element of truth hidden in that misunderstood phrase is, that in a higher state of culture it is found better to disregard the *immediate* technical application in the pursuit of science, and to aim only for the greatest possible perfection and depth in the solution of its individual problems. Whether this is the correct method of procedure and when it is so, is solely a question of the general state of culture. In the early stages of human civilization such a demand is utterly meaningless, and all science is necessarily and naturally confined to immediate life. But the wider and more complex human relations become, the wider and surer must the ability to predict future events become. Then it is the function of prophesy-

ing science to have answers ready for questions which as yet have not become pressing, but which with further development may sooner or later become so.

In the net-like interlacing of the sciences, that is, of the various fields of knowledge, described in the introduction, we must always reckon with the fact that our anticipation of what kind of knowledge we shall next need must always remain very incomplete.. It is possible to foresee future needs in general outline with more or less certainty, but it is impossible to be prepared for particular individual cases which lie on the *border line* of such anticipation, and which may sometimes become of the utmost importance and urgency. Therefore it is one of the most important functions of science to achieve as *perfect* an elaboration as possible of *all* the relations conceivable, and in this practical necessity lies the foundation of the general or *theoretical* elaboration of science.

The Science of Concepts. Here the question immediately arises: how can we secure such perfection? The answer to this general preliminary question of all the sciences belongs to the sphere of the first or the most general of all the sciences, a knowledge of which is presupposed for the pursuit of the other sciences. Since its foundation by the Greek philosopher Aristotle it has borne the name of *logic,* which name, etymologically speaking, hints suspiciously at the *word,* and the word, as is known, steps in where ideas are wanting. Here, however, we have to deal

with the very science of ideas, to which language bears the relation only of a means—and often an inadequate means—to an end. We have already seen how, through the physiologic fact of *memory*, experiences are found in our consciousness which are similar, that is, partially coinciding with one another. These coinciding parts are those concerning which we can make predictions, for the very reason that they coincide in every single instance, and they alone, therefore, constitute that part of our experience which bears results and hence has significance.

4. **Concrete and Abstract.** Such coinciding or repeated parts of similar experiences we call, as already stated, *concepts*. But here, too, attention must immediately be drawn to a linguistic imperfection, which consists in the fact that in such a group of coinciding experiences we designate by the same name both the isolated experience or the object of a special experience and the totality of *all* the coinciding experiences; in other words, all the similar experiences. Thus, *horse* means, on the one hand, quite a definite thing which for the moment forms an object of our experience, and, on the other, the totality of all possible similar objects which have been present in our former experiences, and which we shall meet in our future experiences. It is true that these two sorts of contents of consciousness bearing the same name are distinguished also as *concrete* and *abstract*, and there is an inclination to attribute "reality" only to the first, while the other, as "mere

entities in thought," are relegated to a lesser degree of reality. As a matter of fact, the difference, though important, is of quite another kind. It is the difference between the *momentary experience,* as opposed to the totality of the corresponding *memories* and *expectations.* Hence not so much a difference in *reality* as in *presence.* However, our observations have already made it apparent that presence alone never yields knowledge. A necessary part of knowledge is the memory of former similar experiences. For without such memory and the corresponding comparison, it is quite impossible for us to get at those things which agree and which, therefore, may be predicted; and we should stand before every one of our experiences with the helplessness of a new-born babe.*

5. The Subjective Part. We shall therefore have to recognize realities in abstract ideas in so far as they must rest upon some experiences to be at all intelligible to us. Since the formation of concepts depends upon memories, and these may refer, according to the individual, to very different parts of the same experience of different individuals, concepts always possess an element dependent upon the

* Sometimes on suddenly awaking from a profound sleep a person finds himself for the moment deprived of his personal stock of memories, unable to recall where and in what circumstances he is. No one who has experienced such a condition can ever forget the terrifying sense of helplessness it brings.

individual, or a *subjective* element. This, however, does not consist in the *addition* by the individual of new parts not found in the experience, but, on the contrary, in the different *choice* out of what is found in the experience. If every individual absorbed all parts of the experience, the individual, or subjective, differences would disappear. And since scientific experience endeavors to make the absorption of experiences as complete as possible, it aims nearer and nearer to this ideal by seeking to equalize the subjective deficiency of the individual memory through the collocation of as many and as various memories as possible, thus filling in the subjective gaps in experience as far as possible and rendering them harmless.

6. Empirical Concepts. First and unconditionally those concepts possess reality which always and without exception are based on *experienced* facts. But we can easily make manifold arbitrary combinations of concepts from different experiences, since our memory freely places them at our disposal, and from such a combination we can form a new concept. Of course it is not necessary that our arbitrary combination should also be found in our past or future experiences. On the contrary, we may rather expect that there could be many more arbitrary combinations not to be found in experience than combinations later "confirmed" by experience. The former are purposeless because unreal, the latter, on the contrary, are of the utmost consequence because

upon them is based the real aim of knowledge, prediction. The former are those which have brought the very "reality" of the concepts into ill repute, while the latter show that the formation and the mutual reaction of the concepts practically constitute the entire content of all science. It is of the greatest importance, therefore, to distinguish between the two kinds of concept combinations, and the study of this differentiation forms the content of that most general of all the sciences which we have characterized as logic, or, better, the science of concepts.

7. **Simple and Complex Concepts.** The formation of concepts consists, as we have seen, in the selection of those parts of different but similar experiences which coincide with one another and in the elimination of those that are different in kind. The results of such a procedure may vary greatly according to the number and the difference of the experiences placed in relation with one another. If, for example, we compare only a few experiences, and if, moreover, these experiences are very similar to one another, then the resulting concepts will contain very many parts that agree. But at the same time they will have the peculiarity of not being applicable to other experiences, since these are without some of the coinciding parts of that narrower circle. Thus, for example, the concept which a rustic chained to the soil all his life has of human work does not apply to the work of the city man. A concept will embrace a larger num-

ber of individual cases in proportion as it contains fewer different parts. And by systematically following out this thought we arrive at the conclusion that the concepts that are simple and have no different parts at all find the widest application or are the most general.

The elimination of the non-coinciding parts from the concept-forming experience is called *abstraction.* Obviously abstraction must be carried the farther the more numerous and the more varied the experiences from which the concepts are abstracted, and the simplest concepts are the most abstract.

By looking back over the ground just traversed, the less abstract ideas may also be regarded as the *more complex* in contradistinction to the simpler ones. Only we must guard against the error of literal interpretation and not suppose that the less simple concepts have really been compounded of the simpler ones. In point of origin they actually existed first, since the experience contains the ensemble of all the parts, those which have been retained as well as those which have been eliminated. It is only later, by a characteristic mental operation, after we have analyzed the more complex concept, that is, after we have disclosed the simpler concepts existing in it, that we can compound it again; in other words, execute its synthesis.

These relations bear a striking resemblance to the relations known from chemistry to exist between substances, namely, between elements and

compounds. From the chaos of all objects of experimentation (chemistry purposely limits itself to ponderable bodies) the *pure* substances are sifted out—an operation corresponding to the formation of concepts. The pure substances prove to be either *simple* or *compound*, and the compounds are so constituted that they can each be reduced to a limited number of simple substances. The simple substances, or *elements*, retain this quality of simplicity only until they are recalled; that is, until it has been proved that they, too, can be resolved into still simpler elements. The same is true of the simple concepts. They can claim simplicity only until their complex nature is demonstrated.

With all these similarities we must be extremely careful never to forget the differences existing alongside the agreements. So hereafter we shall make no further use of the chemical simile. It was brought into requisition merely in order to acquaint the beginner the more readily with the entire method of investigation by means of a more familiar field of thought and study. It is quite certain, however, that side by side with the given similarities there are also radical differences. Moreover, the notion of simple and complex concepts or "ideas" had been elaborated by John Locke long before chemistry reached its present state of clearness concerning the concept of the elements.

Nevertheless since then the relation has been completely reversed. While the study of the chemical

elements has in the meantime undergone great development, so that not only have the elements of all the substances coming under the observation of the chemist been discovered, but, inversely, many compound substances have been constructed from their elements, not even an approach to such a development is apparent in the study of concepts. On the contrary, the whole matter has remained at about the same point as that to which John Locke had brought it in the second half of the seventeenth century. This is due above all to the opinion of the most influential philosophers, that Aristotle's logic, or science of concepts, is absolutely true as well as exhaustive and complete, so that, at the utmost, what is left for later generations to do is only to make a change in the form in which the matter is presented. It is true that in more recent times the grave error of this view is beginning to be recognized. We realize that Aristotle's logic embraces but a very small part of the entire field, though in this part he displays the greatest genius. But beyond this general recognition no great step forward has been made. Not even a provisional table of the elementary concepts has been propounded and applied since Locke.

Hence in the following investigation we shall have to speak of the elements or the simpler parts of a complex concept only in the sense that these concept elements are simpler as compared with the complex concepts, but not in the sense that the

simplest or truly elementary concepts have already been worked out. It must be left to later investigators to find these, and it may be expected that the reduction of some concepts until then considered elementary into still simpler ones will take place chiefly in times of great intellectual progress.

Complex concepts can, in the first place, be formed from experience, for in an empirical concept we meet with several conceptual component parts which can be separated from one another by a process of abstraction, but are always found together in the given experiences. For example, the concept *horse* has originated from a very frequent, similarly repeated experience. On analysis it is found to contain a vast number of other concepts, such as quadruped, vertebrate animal, warm-blooded, hairiness, and so on. Horse, then, is obviously a *complex empirical concept.*

On the other hand, we can combine as many simple concepts as we please, even if we did not find them combined in experience, for in reality there is nothing to hinder us from uniting all the concepts provided by memory into any combinations we please. In this way we obtain *complex arbitrary concepts.*

The task of science can now be even more sharply defined than before by the fact that it *permits the construction of arbitrary concepts which in circumstances to be foreseen become empirical concepts.* This is another expression for *prediction,*

which we recognized as the characteristic of science. It goes deeper than the previous definition, because here the means for its realization are given.

8. The Conclusion. First let us consider the scientific import of the complex empirical concepts. It consists in the fact that they accustom us to the coexistence of the corresponding elements of a concept. So that when, in a new experience, we meet with some of these elements together, we immediately suppose that we shall find in the same experience the other elements also which have not yet been ascertained. Such a supposition is called a *conclusion.* A conclusion always exceeds the present experience by predicting an expected experience. Therefore, the form of a conclusion is the universal form of scientific predication.

A conclusion must contain at least two concepts, the one which is experienced, and the one which, on the basis of this experience, is expected. Every complex empirical concept makes such a conclusion possible after it has been separated into simpler concepts. And the simplest case is naturally the one in which there are only *two* parts, or in which only two parts are taken into consideration.

To what extent such a conclusion is valid, that is to say, to what extent the experience produces the anticipated concept, obviously depends upon the reply to a very definite fundamental question. If in experience the union of the two parts of the concept occurs *invariably,* so that one part of the concept is

never experienced unless the other part is also experienced, then there is the *greatest* probability that the expected experience will also have the same character, and that the conclusion will prove valid or true. To be sure, there is no way of making certain that the coincident occurrence of the two concepts, which experience has shown to be *without exception* hitherto, will continue to be so also in the future. For our only means of penetrating into the future consists in applying that conclusion from previous experiences to future experiences, and it can therefore by no means claim absolute validity. There are, however, different *degrees of certainty,* or, rather, *probability,* attaching to such a conclusion. In experiences that occur but rarely the probability is that so far we have experienced only certain combinations of simple concepts, while others, though occurring, have not yet entered within the limited circle of our experience. In such a case a conclusion of the kind mentioned above may be right, but there is also some probability of its being false. On the other hand, in experiences which happen extremely frequently and in the most diverse circumstances, and in which we always find the constant and unexceptional combination, the probability is very strong that we shall find the combination in future experiences also, and the probability of the conclusion approaches practical certainty. Of course, we can never quite exclude the possibility that new relations never as yet experienced might

enter, by which the conclusion which hitherto has always been true would now become false, whether because the expectation entertained prove invalid in single instances or in all cases.

It follows from this that in general, our conclusions will have the greater probability the more generally and the oftener the corresponding experiences have occurred and are occurring. Such concepts as are found consistently in many experiences otherwise different are called *general* concepts, and therefore the probability of the conclusions described will be the greater the more *general* the concepts to which they refer. This obtains to such a degree that we feel that certain very general conclusions must be true always and without exception, and it is "unthinkable" to us that they can ever in any circumstances prove not valid. Such a statement, however, is never anything else than a hidden appeal to experience. For the mere putting of the question, whether the conclusion can also be false, demonstrates that the opposite of what has proved to be the experience so far can be conceived, and the assertion of its "unthinkability" only signifies that such an experience cannot be evoked in the mind by the *memory* for the very reason that, as has been premised, there are no such memories because the experiences did not exist. But since, on the other hand, there is no hindrance to thinking any combinations of concepts at will, we have not the least difficulty, as everybody knows, in thinking any sort

of "nonsense" whatsoever. Only it is impossible to reproduce such combinations from memory.

The scientific conclusion, therefore, first takes the form: if A is, then B is also. Here A and B represent the two simple concepts which are known from experience to be found together in the more complex concept C. The word "is" signifies here some empirical reality corresponding to the concepts. The conclusion may therefore also be expressed, somewhat more circumstantially and more precisely, in this form: if A is experienced, the experience of B is also expected. The evoking of this expectation, which implies its justification, is due to the recollection of the coincidenec of the two concepts in former experiences, and the probability depends, in the manner described above, upon the number of valid cases. Here it must be observed that even individual cases in which our expectations have been deceived do not for the most part lead us to regard the conclusion as generally untrue, that is, to abandon the expectation of B from A. For we know that our experience is always *incomplete,* that in certain circumstances we fail to notice existing factors, and that, therefore, our failure to find that relation valid which, on other occasions, has been found to be valid, may be attributed to subjective causes. In case, however, of the repeated occurrence of such disappointments, we will look elsewhere for relations between these and other elements of experience, in order that thereafter we may

foresee such cases also and include them in our anticipations.

9. **The Natural Laws.** The facts just described have very frequently found expression in the doctrine of the *laws of nature,* in connection with which we have often, as in the man-made social or political laws, conceived of a lawmaker, who, for some reasons, or perhaps arbitrarily, has ordained that things should be as they are and not otherwise. But the intellectual history of the origin of the laws of nature shows that here the process is quite a different one. The laws of nature do not decree what shall happen, but *inform us what has happened and what is wont to happen.* The knowledge of these laws, therefore, makes it possible for us, as I have emphasized again and again, to foresee the future in a certain degree and, in some measure, also to determine it. We determine the future by constructing those relations in which the desired results appear. If we cannot do so either because of ignorance or because of inaccessibility to the required relations, then we have no prospect of fashioning the future according to our desires. The wider our knowledge of the natural laws, that is, of the actual behavior of things, the more likely and more numerous the possibilities for fashioning the future according to our desires. In this way science can be conceived of as the study of how to become happy. For he is happy whose desires are fulfilled.

In this conception the natural laws indicate what

simpler concepts are found in complex concepts. The complex concept *water* contains the simpler ones *liquid*, a certain *density*, *transparency*, *colorlessness*,* and many others. The sentences, water is a liquid, water has a density of one, water is transparent, water is colorless, or, pale blue, etc., are so many natural laws.

Now what predictions do those natural laws enable us to make?

They enable us to predict that when we have recognized a given body as water by virtue of the above properties, we are justified in expecting to find in the same body all the other known properties of water. And so far experience has invariably confirmed such expectations.

Furthermore, we may expect that if in a given specimen of water we discover a relation which up to that time was unknown, we shall find this relation also in all the other specimens of water even though they were not tested for that particular relation. It is obvious how enormously this facilitates the progress of science. For it is only necessary to determine this new relation in some one case accessible to the investigator to enable us to predict the same relation in all the other cases without subjecting them to a new test. As a matter of fact, this is the general method that science pursues. It is this that makes it possible

* More precisely, a very pale blue.

for science to make regular and generally valid progress through the labors of the most various investigators who work independently of one another, and often know nothing of one another.

Of course, it must not be forgotten that such conclusions are always obtained in accordance with the following formula: *things have been so until now, therefore we expect that they will be so in the future.* In every such case, therefore, there is the possibility of error. Thus far, whenever an expectation was not realized, it was almost always possible to find an "explanation" for the error. Either the inclusion of the special case in the general concept proved to be inadmissible because some of its other characteristics were absent, or the accepted characterization of the concept required an improvement (limitation or extension). In other words, one way or another, there was a discrepancy between the concept and the experience, and, as a rule, sooner or later it becomes possible for us to arrive at a better adjustment between them.

This general truth has often been interpreted to mean that in the end such an adjustment must of necessity always be possible to reach, without exception; in other words, that absolutely every part of an experience can be demonstrated as conditioned by natural law. Evidently such an assertion far exceeds the demonstrable. And even the usual conclusion cannot be applied here, that because it has happened so in the past it will happen

so in the future also. For the part of our experiences that we can grasp by natural laws is infinitesimally small in comparison with that in which our knowledge still fails us entirely. I will mention only the uncertainty in predicting the weather for only one day ahead. Moreover, when we consider that until now only the *easiest* problems had been solved, and naturally so, because they were most accessible to the means at hand, then we can readily see that experience offers no basis whatever for such a conclusion. We must not say, therefore, that because we have been able so far to explain all experiences by natural laws it will be so in the future likewise. For we are far from being able to explain all experiences. In fact, it is only a very small part that we have begun to investigate. We are as little justified in saying that we have explained all the problems of our experience that have been subjected to scientific investigation. We have by no means explained all of them. Every science, even mathematics, teems with unsolved problems. So we must resign ourselves to the present status of human knowledge and ability, and may at best express the *hope* founded upon previous experience, that we shall be able to solve more and more of the incalculable number of problems of our experience without indulging in any illusions as to the perfection of this work.

10. The Law of Causation. By reason of its frequency and importance the mental process above de-

scribed has been subjected to the most diverse investigations, and that most general form of the scientific conclusion (which we apply in ordinary life even much more frequently than in science) has been raised, under the name of the law of causation, to a principle anteceding all experience and to the very condition making experience possible. Of this so much is true, that through the peculiar physiological organization of man, *memory in the most general sense*—the easier execution of such processes as have already repeatedly taken place in the organism, as against entirely new kinds of processes—the formation of concepts (of the recurring parts in the constantly changing variety of processes), is especially stimulated and facilitated. By it the recurring parts of experience step into the foreground, and on account of their paramount practical importance for the security of life, it may well be said in the sense of the theory of evolution and adaptation, that the entire structure and mode of life of the organism, especially of the human organism, nay, perhaps life itself, is indissolubly bound up with that foresight and, therefore, with the law of causation also. Of course, there is nothing in the way of calling such a relation an *a priori* relation, if it is so desired. As far as the individual is concerned it no doubt antedates all his experience, since the entire organization which he inherits from his parents had already been formed under such an influence. But that there can be forms or existence

without such an attribute is shown by the whole world of the *inorganic*, in which, as far as our knowledge goes, there is no evidence of either memory or foresight, but only of an immediate passive participation in the processes of the world around them.*

Further, the circumstance that the causal relation is brought about by the peculiar manner in which we react upon our experiences, has sometimes been expressed in this way—the relation of cause and effect does not exist in nature at all, but has been introduced by men. The element of truth in this is, that a quite differently organized being, it is to be supposed, would be able to, or would have to, arrange its experiences according to quite different mutual relations. But since we have no experience of such a being, we have no possibility of forming a valid opinion of its behavior. On the other hand, we must recognize that it is possible, at least formally, to conceive also of kinds of experiences with no coinciding parts, or a world in which there are no experiences at all with coinciding parts. In such, therefore, prediction is impossible. Such a world will not call forth, even in a being endowed with memory, a conception and generalization of the

* It cannot be objected that inorganic nature also is known to be subject to the law of causation. The causal mode of regarding inorganic phenomena is a distinctly human one, and nothing justifies the assertion that the same phenomena cannot be viewed in an entirely different manner.

various experiences in the shape of natural laws. Consequently we must recognize that in addition to the *subjective* factor in the formation of our knowledge of the world, or that factor which is dependent upon our physico-psychical structure, there is also the *objective* character of the world with which we must decidedly reckon, or that character which is independent of us; and that in so far the natural laws contain also objective parts. To represent the relation clearly to our minds by a figure, we may compare the world to a heap of gravel and man to a pair of sieves, one coarser than the other. As gravel passes through the double sieve pebbles of apparently equal size accumulate between the sieves, the larger ones being excluded by the first sieve and the smaller ones allowed to pass by the second. It would be an error to assert that all the gravel consisted of such pebbles of equal size. But it would be equally false to assert that it was the sieves that *made* the pebbles equal.

11. The Purification of the Causal Relation. If by experience we have found a proposition of the content, If A is, then B is also, the two concepts A and B generally consist of several elements which we will designate as a, a′, a″, a‴, etc., and as b, b′, b″, b‴. Now the question arises, whether or not all these elements are essential for the relation in question. It is quite possible, in fact, even highly probable, that at first only a special instance of the existing phenomena was found, that is, that the

concept A, which has been found to be connected with the concept B, contains other determining parts which are not at all requisite to the appearance of B.

The general method of convincing oneself of this is by eliminating one by one the component parts of the concept A, namely, a, a′, a″, etc., and then seeing whether B still appears. It is not always easy to carry out this process of elimination. Our greater or less ability to conduct such investigations depends upon whether we deal with things that are merely the objects of our *observation,* and which we ourselves have not the power to change (as, for example, astronomical phenomena), or with things which are the objects of our *experimentation,* and which we can influence. In the latter case one or another factor is usually found which can be eliminated without the disappearance of B, and then we must proceed in such a way as to form a corresponding new concept A′ from the factors recognized as necessary (which new concept will be more general than the former A), and to express the given proposition in the improved form: If A′ is, then B is also.

Quite similar is the case with the other member of this relation. It often happens that when a, or a″, a‴ is found, somewhat different things appear, which do not fit the concept as first constructed. Then we must multiply the experiences as much as possible in order to determine what constant elements are found in the concept B, and to form from these

constant elements the corresponding concept B′. The improved proposition will then read: if A′ is, then B′ is also.

This entire process may be called the purification of the causal relation. By this term we express the general fact that in first forming such a regular connection, the proper concepts are very seldom brought into relation with one another at once. The cause of it is that at first we make use of *existing* concepts which had been formed for quite a different purpose. It must therefore be regarded as a special piece of good fortune if these old concepts should at once prove suited to the new purpose. Furthermore, the existing concepts are as a rule so vaguely characterized by their names, which we must employ to express the new relation, that for this reason also it is often necessary to determine empirically in what way the concept is to be definitely established.

The various sciences are constantly occupied with this work of the mutual adaptation of the concepts that enter into a causal relation. By way of example, we may take the "self-understood" proposition which we use when we call out to a careless child when it sticks its finger into the flame of a candle, "Fire burns!" We discover that there are self-luminous bodies which produce no increase of temperature, and therefore no sensation of pain. We discover that there are processes of combustion that develop no light, but heat enough to burn one's

fingers. And, finally, the scientific investigation of this proposition arrives at the general expression that, as a rule, chemical processes are accompanied by the development of heat, but that, conversely, such processes may also be accompanied by the absorption of heat. In this way that casual sentence which we call out to the child develops into the extensive science of thermo-chemistry when it is subjected to the continuous purification of the causal relation, which is the general task of science.

It remains to be added that in this process of adapting concepts it is necessary also sometimes to follow the opposite course. This is the case when *exceptions* are noticed in a relation as expressed for the time being; when, therefore, the proposition if A is present, then B is present also, is in a great many instances valid, but occasionally fails. This is an indication that in the concept A an element is still lacking. This element, however, is present in the instances that tally, but absent in the negative cases, and its absence is not noticed because it is not contained in A. Then it is necessary to seek this part, and after it has been found, to embody it in the concept A, which thus passes into the new concept A′.

This case is the obverse of the former one. Here the more suitable concept proves to be less general than the concept accepted temporarily, while in the first case the improved concept is more general. Hence we formulate the rule: exceptions to the

temporary rule require a limitation, while an unforeseen freedom requires an extension, of the accepted concept.

12. Induction. The form of conclusion previously discussed, *because it has been so, I expect it will continue to be so in the future,* is the form through which each science has arisen and has won its real content, that is, its value for the judgment of the future. It is called *inference by induction,* and the sciences in which it is preponderatingly applied are called *inductive sciences.* They are also called experiential or empirical sciences. At the basis of this nomenclature is the notion that there are other sciences, the deductive or rational sciences, in which a reverse logical procedure is applied, whereby from general principles admitted to be valid in advance, according to an absolutely sure logical process, conclusions of like absolute validity are drawn. At the present time people are beginning to recognize the fact that the deductive sciences must give up these claims one by one, and that they already have given them up to a certain extent; partly because on closer study they prove to be inductive sciences, and partly because they must forego the title and rank of a science altogether. The latter alternative applies especially to those provinces of knowledge which have not been used in prophesying the future or cannot be so used.

To return to the inductive method—it is to be noted that *Aristotle,* who was the first to describe it,

proposed two kinds of induction, the *complete* and the *incomplete*. The first has this form: since *all* things of a certain kind are so, each *individual thing* is so. While the incomplete induction merely says: since *many* things of a certain kind are so, *presumably* all things of this kind are so. One instantly perceives that the two conclusions are essentially different. The first lays claim to afford an absolutely certain result. But it rests upon the assumption that *all* the things of the kind in question are known and have been tested as to their behavior. This hypothesis is generally impossible of fulfilment, since we can never prove that there are not more things of the same kind other than those known to us or tested by us. Moreover, the conclusion is *superfluous*, as it merely repeats knowledge that we have already directly acquired, since we have tested *all* the things of the one kind, hence the special thing to which the predication refers.

On the other hand, the *incomplete* induction affirms something that has not yet been tested, and therefore involves as a condition an *extension* of our knowledge, sometimes an extremely important extension. To be sure, it must give up the claim to unqualified or absolute validity, but, to compensate, it acquires the irreplaceable advantage of lending itself to practical application. Indeed, in accordance with the scientific practice justified by experience, described on p. 29, the scientific inductive conclusion

assumes the form: because it has *once* been found to be so, it will *always* be so. From this appears the significance of this method for the enlargement of science, which, without it, would have had to proceed at an incomparably slower pace.

13. **Deduction.** In addition to the inductive method, science has (p. 38) another method, which, in a sense, should be the reverse of the inductive and is claimed to provide absolutely correct results. It is called the *deductive* method, and it is described as the method that leads from premises of general validity by means of logical methods of general validity to results of general validity.

As a matter of fact, there is no science that does or could work in such a way. In the first place, we ask in vain, how can we arrive at such general, or absolutely valid, premises, since all knowledge is of empiric origin and is therefore equipped with the possibility of error as ineradicable evidence of this origin. In the next place, we cannot see how from principles at hand conclusions can be drawn the content of which exceeds that of these principles (and of the other means employed). In the third place, the absolute correctness of such results is doubtful from the fact that blunders in the process of reasoning cannot be excluded even where the premises and methods are absolutely correct. In practice it has actually come to pass that in the so-called deductive sciences doubts and contradictions on the part of the various investigators

of the same question are by no means excluded. To wit, the discussion that has been carried on for centuries, and is not yet ended, over Euclid's parallel theorem in geometry.

If we ask whether, in the sense of the observations we have just made of the formation of scientific principles, there is anything at all like deduction, we can find a procedure which bears a certain resemblance with that impossible procedure and which, as a matter of fact, is frequently and to very good purpose applied in science. It consists in the fact that general principles which have been acquired through the ordinary incomplete induction *are applied to special instances which, at the proposition of the principle, had not been taken into consideration,* and whose connection with the general concept had not become directly evident. Through such application of general principles to cases that have not been regarded before, specific natural laws are obtained which had not been foreseen either, but which, according to the probability of the thesis and the correctness of the application are also probably correct. However, the investigator, bearing in mind the factor of uncertainty in these ratiocinations feels in each such instance the need for testing the results by experience, and he does not consider the *deduction* complete until he had found *confirmation* in experience.

Deduction, therefore, actually consists in the searching out of particular instances of a principle

established by induction and in its confirmation by experience. This conduces to the growth of science, not in breadth, but in profundity. I again resort to the comparison I have frequently made of science with a very complex network. At first glance we cannot obtain a complete picture of all the meshes. So, at the first proposition of a natural law an immediate survey of the entire range of the possible experiences to which it may apply is inachievable. It is a regular, important, and necessary part of all scientific work to learn the extent of this range and investigate the specific forms which the law assumes in the remoter instances. Now, if an especially gifted and far-seeing investigator has succeeded in stating in advance an especially general formulation of an inductive law, it is everywhere confirmed in the course of the trial applications, and the impression easily arises that confirmation is superfluous, since it results simply in what had already been "deduced." In point of fact, however, the reverse is not infrequently the case, that the principle is *not* confirmed, and conditions quite different from those anticipated are found. Such discoveries, then, as a rule, constitute the starting-point of important and far-reaching modifications of the original formulation of the law in question.

As we see, deduction is a necessary complement of, in fact, a part of, the inductive process. The history of the origin of a natural law is in general

as follows. The investigator notices certain agreements in individual instances under his observation. He assumes that these agreements are general, and propounds a temporary natural law corresponding to them. Then he proceeds by further experimentation to test the law in order to see whether he can find full confirmation of it by a number of other instances. If not, he tries other formulations of the law applicable to the contradictory instances, or exclusive of them, as not allied. Through such a process of adjustment he finally arrives at a principle that possesses a certain range of validity. He informs other scientists of the principle. These in their turn are impelled to test other instances known to them to which the principle can be applied. Any doubts or contradictions arising from this again impel the author of the principle to carry out whatever readjustments may have become necessary. Upon the scientific imagination of the discoverer depends the range of instances sufficing for the formulation of the general inductive principle. It also frequently depends upon conscious operations of the mind dubbed "scientific instinct." But as soon as the principle has been propounded, even if only in the consciousness of the discoverer, the deductive part of the work begins, and the consequent test of the proposition has the most essential influence on the value of the result.

It is immediately evident that this *deductive* part is of all the more weight, the more *general* the con-

cepts in question are. If, in addition, the inductive laws posited soon prove to be of a comparatively high degree of perfection, we obtain the impression described above, that an unlimited number of independent results can be deduced from a premise. *Kant* was keenly alive to the peculiarity of such a view, which had been widely spread pre-eminently by *Euclid's* presentation of geometry, and he gave expression to his opinion of it in the famous question: *How are a priori judgments possible?* We have seen that it is not always a question of *a priori* judgments, but also of inductive conclusions applied and tested according to deductive methods.

14. Ideal Cases. Each experience may generally be considered under an indefinite number of various concepts, all of which may be abstracted from that experience by corresponding observations. Accordingly an indefinite number of natural laws would be required for prophesying that experience in all its parts. Likewise the indefinite number of premises must be known through the application of which those natural laws acquire a certain content. Thus it seems as if it were altogether impossible to apply natural laws for the determination of a single experience to come, and in a certain sense this is true (p. 30). For example, when a child is born, we are quite incapable of foretelling the peculiar events that will occur in its life. Beyond the statement that it will live a while and then die, we can make only

the broadest assertions qualified by numerous "ifs" and "buts."

If, in spite of this, we arrange a very great part of our life and activity according to the prophecies we make in regard to numerous details in life, basing them upon natural laws, the question arises, how we get over the difficulty, or, rather, the impossibility just referred to.

The answer is, that we repeatedly so find or can form our experiences that certain natural relations *preponderatingly* determine the experience, while the other parts that remain undetermined fall into the background. *The prophecy will cover so considerable a part of the experience that we can forego previous knowledge of the rest.* We can foretell enough to render a practical construction of life possible, and increasing experience, whether the personal experience of the individual or the general experience of science, constantly enlarges this controllable part of future experiences.

The procedure of science is similar to that of practical life, though freer. Whenever an investigator seeks to test a natural law of the form: if A is so, then B is so, he endeavors to choose or formulate the experiences in such a way that the fewest possible extraneous elements are present, and that those that are unavoidable should exert the least possible influence upon the relation in question. He never succeeds completely. In order, nevertheless, to reach a conclusion as to the form the rela-

tion will take without extraneous influences, the following general method is applied.

A series of instances are investigated which are so adjusted that the influence of the extraneous elements grows less and less. Then the relation investigated approaches a limit which is never quite reached, but to which it draws nearer and nearer, the less the influence of the extraneous elements. And the conclusion is drawn that if it were possible to exclude the extraneous elements entirely, the limit of the relation would be reached.

A case in which none of the extraneous elements of experience operate is called an *ideal case,* and the inference from a series of values leading to the limit-value is an *extrapolation*. *Such extrapolations to the ideal case* are a quite natural procedure in science, and a very large part of natural laws, especially all quantitative laws, that is, such as express a relation between measurable values, have precise validity only in ideal cases.

We here confront the fact that many natural laws, and among them the most important, are expressed as, and taken to be, conditions *which never occur in reality*. This seemingly absurd procedure is, as a matter of fact, the best fitted for scientific purposes, since ideal cases are to be distinguished by this, *that with them the natural laws take on the simplest forms*. This is the result of the fact that in ideal cases we intentionally and arbitrarily overlook every complication of the determining factors,

and in describing ideal cases we describe the simplest conceivable form of the class of experiences in question. The real cases are then constructed from the ideal cases by representing them as the sum of all the elements that have an influence on the experience or the result. Just as we can represent the unlimited multitude of finite numbers by the figures up to ten, so we can represent an unlimited quantity of complicated events by a finite number of natural laws, and so reach a highly serviceable approximation to reality.

Thus geometry deals with absolutely straight lines, absolutely flat surfaces, and perfect spheres, though such have never been observed, and the results of geometry come the closer to truth, the more nearly the real lines, surfaces, and spheres correspond to the ideal demands. Similarly, in physics, there are no ideal gases or mirrors, or in chemistry ideally pure substances, though the expressed simple laws in these sciences are valid for only such bodies. The non-ideal bodies of these sciences, which reality presents in various degrees of approximation, correspond the more closely to these laws, the slighter the deviation of the real from the ideal. And the same method is applied in the so-called mental sciences, psychology and sociology, in which the "normal eye" and a "state with an entirely closed door" are examples of such idealized limit-concepts.

15. **The Determinateness of Things.** A very widespread view and a very grave one, because of

its erroneous results, is *that by the natural laws things are unequivocally and unalterably determined down to the very minutest detail.* This is called *determinism,* and is regarded as an inevitable consequence of every natural scientific generalization. But an accurate investigation of actual relations produces something rather different.

The most general formulation of the natural law: *if A is experienced, then we expect B,* necessarily refers in the first place only to certain *parts* of the thing experienced. For perfect similarity in two experiences is excluded by the mere fact that we ourselves change unceasingly and one-sidedly. Consequently, no matter how accurate the repetition of a former experience may be, our very participation in it, an element bound to enter, causes it to be different. Therefore we deal with only a *partial* repetition of any experience, and the common part is all the smaller a fraction of the entire experience, the more *general* the concept corresponding to this part. But the most general and most important natural laws apply to such very general ideas, and accordingly they determine only a small part of the whole result. Other parts are determined by other laws, but we can never point out an experience that has been determined completely and unequivocally by natural laws known to us. For example, we know that when we throw a stone, it will describe an approximate parabolic curve in falling to the ground. But if we should attempt to determine its

course accurately, we should have to take into consideration the resistance of the air, the rotatory motion of the stone upon being thrown, the movement of the earth, and numerous other circumstances, the exact determination of which is a matter beyond the power of all sciences. Nothing but an *approximate* determination of the stone's course is possible, and every step forward toward accuracy and absoluteness would require scientific advances which it would probably take centuries to accomplish.

Science, therefore, can by no means determine the exact linear course that the stone will take in its fall. It can merely establish a certain broader path within which the stone's movement will remain. And the path is the wider the smaller the progress science has made in the branch in question. The same conditions prevail in the case of every other prediction based upon natural laws. Natural laws merely provide a certain frame within which the thing will remain. But which of the infinitely numerous possibilities within this frame will become reality can never be absolutely determined by human powers.

The belief that it is possible has been evoked merely by a far-reaching method of abstraction on the part of science. By assuming in place of the stone "a non-extended point of mass" and by disregarding all the other factors which in some way (whether known or unknown) exercise an influence

on the stone's movement, we can effect an apparently perfect solution of the problem. But the solution is not valid for real experience, merely for an ideal case, which bears only a more or less profound similarity to the real. It is only such an ideal world, that is, a world arbitrarily removed from its actual complexity, that has the quality of absolute determinateness which we are wont to ascribe to the real world.

We might point to the method of abstraction generally adopted in science and to the extrapolation to ideal cases which has just been explained, and regard the assertion of the absolute determinateness of events in the world as a justified extrapolation to the ideal case. In other words, we might say that we know all the natural laws and how to apply them perfectly to the individual instances. In controversion of this it must be said that the ulterior justification of such ideal extrapolation is not yet feasible. The justification lies in the demonstration that the real cases approximate the ideal the more closely the more we actualize our presumptions. But in this case this is not feasible, since, for the greater part of our experiences, we do not even know the approximate or ideal natural laws by the help of which we can construct such ideal cases. For instance, the whole province of organic life is at present essentially like an unknown land, in which there are only a few widely separated paths ending in *culs-de-sac*.

16. The Freedom of the Will. This relation explains why, on the one hand, we assume a far-reaching determinateness for many things, that is, for all those accessible to scientific treatment and regulation, and why, on the other hand, we have the consciousness of acting *freely*, that is, of being able to control future events according to the relations they bear to our wishes. Essentially there is no objection to be found to a fundamental determinism which explains that this feeling of freedom is only a different way of saying *that a part of the causal chain lies within our consciousness,* and that we feel these processes (in themselves determined) as if we ourselves determined their course. Nor can we prove this idea to be false, that, since the number of factors which influence each experience is indefinitely great and their nature indefinitely complex, each event would appear to be determined in the eyes of an all-comprehensive intellect. But to our finite minds an undetermined residue necessarily remains in each experience, and to that extent the world must always remain in part practically undetermined to human beings. Thus, both views, that the world is not completely determined, and that it really is, though we can never recognize that it is, lead practically to the same result: *that we can and must assume in our practical attitude to the world that it is only partially determined.*

But if two different lines of thought in the whole

world of experience everywhere lead to the same result, they cannot be materially, but merely formally or superficially, different. For those things are alike which cannot be distinguished. There is no other definition of alikeness. Thus, if we see that the age-long dispute between these two views always breaks out afresh without seeming to be able to reach an end, this is readily understood, from what has been said, since the very same essential arguments which can be adduced of *one* view can be used as a prop for the *other* view, because in their essential results the two are the same. I have discussed this matter because it presents a very telling example of a method to be applied in all the sciences when dealing with the solution of old and ever recurrent moot questions. Each time we encounter such problems, we must ask ourselves: what would be the difference empirically if the one or the other view were correct? In other words, we first assume the one to be correct, and develop the consequences accordingly. Then we assume the second to be correct and develop the consequences accordingly. If in the two cases the consequences differ in a certain definite point, we at least have the possibility of ascertaining the false view by investigating in favor of which case experience decides on this point. However, we may not conclude that by this the other view has been proved to be entirely correct. It likewise may be false, only with the peculiar quality that in the case in ques-

tion it leads to the correct conclusions. That such a thing is possible, every one knows who has attentively observed his own experiences. How often we act correctly in actual practice, though we have started out on false premises! The explanation of this possibility resides in the highly composite nature of each experience and each assumption. It is quite possible—and, in fact, it is the general rule—that a certain view contains true elements, but *along with them false elements also.* In applications of the view where the true elements are the decisive factors, true results are obtained, despite the errors present. Likewise, false results will be achieved where the false elements are decisive, despite the true results that can be had, or have been had, elsewhere, by means of the true elements. Hence, in case of the "confirmation," we can only conclude that that portion of the view essential for the instance in question is correct.

One readily perceives that these observations find application in all provinces of science and life. There are no absolutely correct assertions, and even the falsest may in some respect be true. There are only greater and lesser probabilities, and every advance made by the human intellect tends to increase the degree of probability of experiential relations, or natural laws.

17. **The Classification of the Sciences.** From the preceding observations the means may be drawn for outlining a complete table of the sciences. How-

ever, we must not regard it complete in the sense that it gives every possible ramification and turn of each science, but that it sets up a frame inside of which at given points each science finds its place, so that, in the course of progressive enlargement, the frame need not be exceeded.

The basic thought upon which this classification rests is that of graded abstraction. We have seen (p. 19) that a concept is all the more general, that is, is applicable to all the more experiences, the fewer parts or elementary concepts it contains. So we shall begin the system of the sciences with the most general concepts, that is, the elementary concepts (or with what for the time being we shall have to consider elementary concepts), and, in grading the concept complexes according to their increasing diversity, set up a corresponding graded series of sciences. One thing more is to be noted here, that this graded series, on account of the very large number of new concepts entering, must produce a correspondingly great number of diverse sciences. For practical reasons groups of such grades have been combined temporarily. Thereby a rougher classification, though one easier to obtain a survey of, has been made. The most suitable and lasting scheme of this sort was originated by the French philosopher, *Auguste Comte,* since whom it has undergone a few changes.

Below is the table of the sciences, which I shall then proceed to explain:

I. *Formal Sciences.* Main concept: order
- Logic, or the science of the Manifold
- Mathematics, or the science of Quantity
- Geometry, or the science of Space
- Phoronomy, or the science of Motion

II. *Physical Sciences.* Main concept: energy
- Mechanics
- Physics
- Chemistry

III. *Biological Sciences.* Main concept: life
- Physiology
- Psychology
- Sociology

As is evident, we first have to deal with the three great groups of the formal, the physical, and the biological sciences. The formal sciences treat of characteristics belonging to all experiences, characteristics, consequently, that enter into every known phase of life, and so affect science in the broadest sense. In order immediately to overcome a widespread error, I emphasize the fact that these sciences are to be considered just as experiential or empirical as the sciences of the other two groups, as to which there is no doubt that they are empirical. But because the concepts dealt with by the first group are so extremely wide, and the experiences corresponding to them, therefore, are the most general of all experiences, we easily forget that we are dealing with experiences at all; and our very firmly rooted consciousness of the unqualified similarity of these ex-

periences causes them to seem native qualities of the mind, or *a priori* judgments. Nevertheless, mathematics has been proved to be an empirical science by the fact that in certain of its branches (the theory of numbers) laws are known which have been found empirically and the "deductive" proof of which we have as yet not succeeded in obtaining. The most general concept expressed and operative in these sciences is the concept of order, of *conjugacy* or *function,* the content and significance of which will become clear later in a more thorough study of the special sciences.

In the second group, the physical sciences, the arbitrariness of the classification becomes very apparent, since these sciences are among the best known. We are perfectly justified in regarding mechanics as a part of physics; and in our day physical chemistry, which in the last twenty years suddenly developed into an extended and important special science, thrust itself between physics and chemistry.

The most general concept of the physical sciences is that of *energy,* which does not appear in the formal sciences. To be sure it is not a fundamental concept. On the contrary, its characteristic is undoubtedly that of compositeness, or, rather, complexity.

The third group comprehends all the relations of living beings. Their most general concept, accordingly, is that of *life.* By physiology is understood the entire science dealing with non-psychic life

phenomena. It therefore embraces what is called, in the present often chance arrangement of scientific activities, botany, zoology, and physiology of the plants, animals, and man. Psychology is the science of mental phenomena. As such, it is not limited to man, even though for many reasons he claims by far the preponderating part of it for himself. Sociology is the science which deals with the peculiarities of the human race. It may therefore be called anthropology, but in a far wider sense than the word is now applied.

18. The Applied Sciences. It will be remarked that the grouping of the table gives no place at all in its scheme to certain branches of learning taught in the universities and equally good technical institutions. We look in vain not only for theology and jurisprudence, but also for astronomy, medicine, etc.

The explanation and justification of this is, that for purposes of systematization we must distinguish between *pure* and *applied* sciences. By virtue of their strictly conceptual exclusiveness the pure sciences constitute a regular hierarchy or graded series, so that all the concepts that have been used and dealt with in the preceding sciences are repeated in the following sciences, while certain characteristic new concepts enter in addition. Thus logic, the science of the manifold, exercises its dominion over all the other sciences, while the specific concepts of physics and chemistry have nothing to do with it, though

they are of importance to all the biologic sciences. Through this graded addition of new (naturally empiric) concepts, the construction of the pure sciences proceeds in strict regularity, and their problems arise exclusively from the application of new concepts to all the earlier ones. In other words, their problems do not reach them accidentally from without, but result from the action and reaction of their concepts upon one another.

At the same time there are problems that each day sets before us without regard to system. These come from our endeavor to improve life and avert evil. In the problems of life we are confronted by the whole variety of possible concepts, and under the day's immediate compulsion we cannot wait, if we are sowing crops or helping a sick man, until physiology and all the other appropriate sciences have solved all the problems of plant growth and the changes of the human body and human energy. When other signs fail, we use the position of the stars for finding our way on the high seas. In this manner we turn the teaching of the stars, or astronomy, into an applied science, in which at first mechanics alone seemed to have a part. Later physics took a share in it, then optics took a particularly prominent share, and in recent times not only did chemistry find its way into astronomy, but the specifically biologic concept of evolution was applied in astronomy with success.

Thus, side by side with the pure sciences are the

applied, which are to be distinguished from the pure sciences by the fact that they do not unfold their problems systematically, but are assigned them by the external circumstances of man's life. The pure sciences, therefore, almost always have a larger or smaller share in the tasks of the applied sciences. For instance, in building a bridge or railroad, physical problems have to be taken into consideration as well as sociologic problems (problems of trade), and a good physician should be a psychologist as well as a chemist.

But since all the individual questions arising in the applied sciences may be considered essentially as problems of one or other pure science, they need not be explicitly enumerated along with the pure sciences, especially since their development is greatly dependent upon temporary conditions and is therefore incapable of simple systematization.

PART II

LOGIC, THE SCIENCE OF THE MANIFOLD, AND MATHEMATICS

19. **The Most General Concept.** If we try to conceive the whole structure of science according to the principle of the increasing complexity of concepts, the first question which confronts us is, What concept is the *most general* of all possible concepts, so general that it enters into every concept formation and acts as a decisive factor? In order to find this concept let us go back to the psycho-physical basis of concept formation, namely, *memory,* and let us investigate what is the general characteristic determining memory. We soon perceive that if a being were to lead an absolutely uniform existence, *no* memories could be evoked. There would be nothing by which the past could be distinguished from the present, hence nothing by which to compare them. So the "primal phenomenon" of conscious thought is the realization of a *difference,* a difference between memory and the present, or, to put the same idea still more generally, between two memories.

Our experiences, therefore, are divided into two

parts, distinguished from each other. In order to predicate something of a perfectly general nature concerning those parts, without regard to their particular content, we must, in accordance with the means employed in human intercourse, designate them by a *name.* Now in all human languages there is a great deal of arbitrariness and indefiniteness in the relations between the concepts and the names applied to them, which render all accurate work in the study of concepts extremely difficult. It is necessary, therefore, to state definitely in each particular instance with what conceptual content a given name is to be connected. Every experience in so far as it is differentiated from other experiences we shall call simply an *experience* without making a distinction between a so-called inner or outer experience.

Many of the experiences remain isolated, because they are not repeated in a similar form, and so do not remain in our memory. They depart from our psychic life once for all and leave no further consequences or associations. But some experiences recur with greater or less uniformity, and become permanent parts of psychic life. Their duration is by no means unlimited. For even memories fade and disappear. However, they extend through a considerable part of life, and that suffices to give them their character.

The aggregate of similar experiences, hence of experiences conceptually generalized, we shall call *things. A thing, therefore, is an experience which*

has been repeated, and is "recognized" by us. That is, it is felt as repeated and conceptually comprehended. In other words, all experiences of which we have formed concepts are things, and *the concept of thing itself is the most general concept,* since, according to its definition, it includes all possible concepts. Its "essence," or determining characteristic, lies in the possibility of differentiating any one thing from another. Things we do not differentiate we call *the same,* or *identical.* Here we shall leave undecided the question whether this lack of differentiation occurs because we *cannot,* or because we *would not,* differentiate. All experiences generalized into one concept are therefore felt or regarded as the same in reference to this concept. Now, since concepts arise unconsciously as well as consciously, the first is a case of identities which had been directly felt as such. On the other hand, in the second case, the process is that of consciously disregarding or abstracting the existing differences in order to form a concept into which these do not enter. This last process is applied in the highest degree possible in obtaining the concept *thing.*

20. Association. The experience of the *connection* or *relation* between various things is also derived from the nature of our experiences in the most general sense. When we recall a thing A, another thing B comes to our mind, the memory of which is called forth by A, and *vice versa.* The cause of this invariably lies in some experiences in

which A and B occur together. In fact, A and B must have occurred together a number of times. Otherwise they would have disappeared from memory. In other words, it is the fact of the *complex concept* which appears in such connections between various things. Two things, A and B, which are connected with each other in such a way, are said to be associated. Association in the most general sense means nothing more than that when we think of B we also have A in our consciousness, and *vice versa*. However, we can at will make the association more definite, so that quite definite thoughts or actions will be connected with the association of B. These thoughts and actions are then the same for all the individual cases occurring under the concept A and B.

If we associate with the thing B another thing C, we obtain a relation of the same nature as that obtained by the association of A and B. But at the same time a new relation arises which was not directly sought, namely, the association of A to C. If A recalls B, and B recalls C, A must inevitably recall C also. This psychologic law of nature is productive of numberless special results. For we can apply it directly to still another case, the association of a fourth thing D to the thing C, whereby new relations are necessarily established also between A and D as well as between B and D. By positing the *one* relation C : D there arise two new relations not immediately given, namely, A : D and

B : D. The reason the other relations arise is because C was not taken free from all relations, but had already attached to it the relations to A and B. These relations of C, therefore, brought A and B into the new relation with D.

By this simplest and most general example we recognize the type of the deductive process (p. 41), namely, the discovery of relations which, it is true, have already been established by the accepted premises, but which do not directly appear in undertaking the corresponding operations. In the present case, to be sure, the deduction is so apparent that the recognition of the relations in question offers not the slightest difficulty. But we can easily imagine more complicated cases in which it is much more difficult to find the actually existing relations, and so in certain circumstances we may search for them a long time in vain.

21. The Group. The aggregate of all individual things occurring in a definite concept, or the common characteristics of which make up this concept, is called a group. Such a group may consist of a limited or finite number of members, or may be unlimited, according to the nature of the concepts that characterize it. Thus, all the integers form an unlimited or infinite group, while the integers between ten and one hundred (or the two-digit numbers) form a limited or finite group.

From the definition of the group concept follows the so-called classic *process of argumentation* of the

syllogism. Its form is: *Group A is distinguished by the characteristic of B. The thing C belongs to group A. Therefore C has the characteristic of B.* The prominent part ascribed by *Aristotle* and his successors to this process is based upon the *certainty* which its results possess. Nevertheless, it has been pointed out, especially by *Kant,* that judgments or conclusions of such a nature (which he called analytic) have no significance at all for the progress of science, since they express only what is already known. For in order to enable us to say that the thing C belongs to group A, we must already have recognized or proved the presence of the group characteristic B in C, and in that case the conclusion only repeats what is already contained in the second or minor premise.

This is evident in the classic example: All men are mortal. Caius is a man. Therefore Caius is mortal. For if Caius's mortality were not known (here we are not concerned how this knowledge was obtained), we should have no right to call him a man.

At the same time the character of the really scientific conclusion based upon the incomplete induction becomes clear. It proceeds according to the following form. The attributes of the group A are the characteristics of a, b, c, d. We find in the thing C the characteristics a, b, c. Therefore we presume that the characteristic d will also be found in C. The ground for this presumption is that we

have learned by experience that the characteristics mentioned have always been found together. It is for this reason, and for this reason only, that we may assume from the presence of a, b, c the presence of d. In the case of an arbitrary combination, in which it is possible to combine other characteristics, the conclusion is unfounded. But if, on the other hand, the formation of the concept A with the characteristics of a, b, c, d has been caused by repeated and habitual experience, then the conclusion is well founded; that is, it is probable.

As a matter of fact, however, that classic example which is supposed to prove the absolute certainty of the regular syllogism turns out to be a hidden inductive conclusion of the incomplete kind. The premise, Caius is a man, is based on the attributes a, b, c (for example, erect bearing, figure, language), while the attribute d (mortality) cannot be brought under observation so long as Caius remains alive. In the sense of the classic logic, therefore, we are not justified in the minor premise, Caius is a man, while Caius is alive. The utter futility of the syllogism is apparent, since, according to it, it is only of dead men that we can assert that they are mortal.

From these observations it becomes further apparent that logic, whether it is the superfluous classic logic or modern effective inductive logic, is nothing but a part of the group theory, or science of manifoldness, which appears as the first, because it is

the most general member of the mathematical sciences (this word taken in its widest significance). But according to the hierarchic system in harmony with which the scheme of all the sciences had been consciously projected, we cannot expect anything else than that those sciences which are needful for the pursuit of all other sciences (and logic has always been regarded as such an indispensable science, or, at least, art) should be found collected and classified in the first science.

22. **Negation.** When the characteristics a, b, c, d of a group have been determined, then the aggregate of all things existing can be divided into two parts, namely, the things which belong to the group A and those which do not belong to it. This second aggregate may then be regarded as a group by itself. If we call this group "not-A," it follows from the definition of this group that the two groups, A and not-A, together form the aggregate of all things.

This is the meaning and the significance of the linguistic form of *negation*. It excludes the thing negated from any group given in a proposition, and this relegates it to the second or complementary group.

The characteristic of such a group is the common absence of the characteristics of the positive group. We must note here that the absence of even *one* of the characteristics a, b, c, d excludes the incorporation of the thing into the group A, while the mere

absence of this characteristic suffices to include it in the group not-A. We can therefore by no means predicate of group not-A that each one of its members must lack *all* the characteristics a, b, c, d. We can only say that each of its members lacks at least one of the characteristics, but that one or some may be present, and several or all may be absent. From this follows a certain asymmetry of the two groups, which we must bear in mind.

The consideration of this subject is especially important in the treatment of negation in the conclusions of formal logic. As we shall make no special use of formal logic, we need not enter into it in detail.

23. Artificial and Natural Groups. The combination of the characteristics which are to serve for the definition of a group is at first purely arbitrary. Thus, when we have chosen such an arbitrary combination, a, b, c, d, we can eliminate one of the characteristics, as, for example, c, and form a group with the characteristics a, b, d. Such a group, which is *poorer in characteristics,* will, in general, be *richer in members,* for to it belong, in the first place, all the things with the characteristics a, b, c, d, of which the first group consisted, and in addition all the things which, though not possessing c, possess a, b, and d.

If we call such groups related as contain common characteristics, though containing them in different members and combinations, so that the definition of

the one group can be derived from the other by the elimination or incorporation of individual characteristics, then we can postulate the general thesis *that in related groups those must be richer in members which are poorer in characteristics, and inversely*. This is the precise statement of the proposition of the less definite thesis stated above.

For the purposes of systematization we have assumed that we can arbitrarily eliminate one or another characteristic of a group. In experience, however, this often proves inadmissible. As a rule we find that the things which lack one of the characteristics of a group will also lack a number of other characteristics; in other words, that the characteristics are not all independent of one another, but that a certain number of them go together, so that they are present in a thing either in common or not at all.

This case, however, can be referred to the general one first described, by treating the characteristics belonging together as being *one* characteristic, so that the group is defined solely by the independent characteristics. Then, according to the definition, we can, without losing our connection with experience, carry out that formal manifoldness of all possible related groups which yields what is called a *classification* of the corresponding things.

If for the determination of a group a definite number of independent characteristics is taken, say, a, b, c, d, and e, then we have at first the narrowest

or poorest group abcde. By the elimination of one characteristic we obtain the five groups, bcde, acde, abde, abce, and abcd. If we omit one other characteristic we get ten different groups abc, abd, abe, acd, ace, ade, bcd, bce, bde, cde. Likewise, there are ten groups with two characteristics each, and finally five groups with one characteristic each. All these groups are related. There is a science, the Theory of Combinations, which gives the rules by which, in given elements or characteristics, the kind and number of the possible groups can be found. The theory of combinations enables us to obtain a complete table and survey of all possible complex concepts which can be formed from given simple ones (whether they be really elementary concepts, or only relatively so). When in any field of science the fundamental concepts have been combined in this manner, a complete survey can be had of all the possible parts of this science by means of the theory of combinations.

In order to present this process vividly to our minds, let us take as an example the science of the chemical combination of substances which form an important part of chemistry. There are about eighty elements in chemistry, and this science has to treat of

a) each of the eighty elements by itself
b) all substances containing two elements and no more
c) all substances containing three elements

d, e, f, etc.) the substances containing four, five, and six, etc., elements,

until finally we reach a group (not existing in experience) embracing substances formed of *all* the elements. That there is no such substance in the present scope of human knowledge has, of course, no significance for the structure of the scheme. What is significant is the fact that the scheme really embraces and arranges all possible substances in such a way that we cannot conceive of any case in which a newly discovered substance cannot after examination immediately be classed with one of the existing groups.

To cite an example from another science. Physics, it will be recalled, may be considered to be the science of the different kinds of energy. This science, accordingly, is divided first into the study of the properties of each energy, and then into the study of the relations of two energies, of three energies, of four energies, etc. Here, too, we may say that in the end there can be no physical phenomenon which cannot be placed in one of the groups so obtained.

Of course, neither in chemistry nor in physics does this mean that each *new* case will fall within the scheme obtained by the exhaustive combination of elementary concepts (whether chemical elements or kinds of energy) *known* at the time. It is quite possible that a new thing under investigation contains a *new* elementary concept, so that on account

of it the scheme must be enlarged through the embodiment of this new element. But simultaneously a corresponding number of new groups appear in the scheme, and the investigator's attention is directed to the fact that he still has a reasonable prospect, in favorable circumstances, of discovering these new things also. Thus combinatory schematization serves not only to bring the existing content of science into such order that each single thing has its assigned place, but the groups which have thereby been found to be vacant, to which as yet nothing of experience corresponds, also point to the places in which science can be completed by new discoveries.

From the above presentation it is apparent how from the two concepts "thing" and "association" alone a great manifoldness of various and regular forms can be developed. They are purely empirical relations, for the fact that several things can be combined in the graded series described above according to a fixed rule does not follow merely from the two concepts, but must be *experienced*. But, on the other hand, both concepts are so general that the experiences obtained in some cases can be applied to all possible experiences and may serve the purpose of classifying and making a general survey of them.

The above statements, however, have by no means exhausted the possibilities. For it has been tacitly assumed that in the combination of several things the *sequence* according to which this combination

takes place should not condition a difference of the result. This is true of a number of things, but not of all. In order, therefore, to exhaust the possibilities the theory of combinations must be extended also to cases in which the sequence is to be taken account of, so that the form ab is regarded as different from ba.

We will not undertake to work out the results of this assumption. It is obvious that the manifoldness of the various cases is much greater than if we neglect the sequence. On this point we have one more observation to make, that further causes for diversity exist. It is true that a chemical combination is not influenced by the sequence in which its elements enter the combination, but there do occur with the same elements differences in their *quantitative relations,* and thereby a new complexity is introduced into the system, so that two or more similar elements can form different combinations according to the difference in the quantitative relations. Still, even with this, the actual manifoldness is not exhausted, for from the same elements and with the same quantitative relations there can arise different substances called *isomeric,* which, for all their similarity, possess different energy contents. But the first scheme is not demolished, nor does it become impracticable because of this increase of manifoldness. What simply happens is that *several* different things instead of one appear in the same group of the original scheme, the systematic

classification of which necessitates a further schematization by the use of other characteristics.

24. **Arrangement of the Members.** Since we have started from the proposition that all members of a group are different from one another, we have perfect liberty to arrange them. The most obvious arrangement according to which some *one* definite member is followed by a *single* other member and so forth (as, for example, the arrangement of the letters of the alphabet) is by no means the only mode of arrangement, though it is the simplest. Besides this *linear* arrangement, there is also, for instance, the one in which two new members follow simultaneously upon each previous one, or the members may be disposed like a number of balls heaped up in a pyramid. However, we shall not have much occasion to occupy ourselves with these complex types of arrangement, and can therefore limit our considerations at first to the simplest, that is, to the linear arrangement.

This simplest of all possible forms expresses itself in the fact *that the immediately experienced things of our consciousness are arranged in this way.* In point of fact, the contents of our consciousness proceed in linear order, one single new member always attaching itself to an existing member. This law, however, is not strictly and invariably adhered to. It sometimes happens that our consciousness continues for a while to pursue the direction of thought it has once taken, although a branching off had al-

ready taken place at a former point, at which a new chain of thought had begun. Nevertheless, one of these chains usually breaks off very soon, and the linear character of the inner experience is immediately restored. Of certain specially powerful intellects it is recorded that they could keep up several lines of thought for a considerable length of time—Julius Cæsar, for instance.

The biologic peculiarity here mentioned of the linear juxtaposition of the contents of our consciousness has led to the concept of *time,* which has been appropriately called a *form of inner life.* That all our experiences succeed each other in time is equivalent to saying that our thought processes represent a group in linear arrangement. As appears from the above observations, this is by no means an absolute form, unalterable for all times. On the contrary, a few highly developed individuals have already begun to emancipate themselves from it. But the existing form is so firmly fixed through heredity and habit that it still seems impracticable for most men to imagine the succession of the inner experiences in a different way than by a line or by *one dimension.* Since, on the other hand, we have all learned to feel space as *tri-dimensional,* although optically it appears to possess only two dimensions (we see length and breadth, and only infer thickness from secondary characteristics), we come to recognize that the linear form by which we represent the succession of our experiences is a matter

of adaptation, and that because the change has been extremely slight in the course of centuries it produces the impression of being unalterable.*

These discussions lead to a further difference that can exist in groups of linear arrangement. While in the first example we chose, the alphabet, the sequence was quite *arbitrary*, since any other sequence is just as possible, the same cannot be said of experiences into which the element of time enters. These are not arbitrary, but are arranged by special circumstances depending upon the aggregate of things which co-operate in the given experiences.

While, therefore, a group with free members, that is, members not determined in their arrangement by special circumstances, can be brought into linear order in very different ways, there are groups in which only one of those orders actually occurs. We see at once that in free groups the number of different orders possible is the greater, the greater the group itself. The theory of combinations teaches how to calculate these numbers which play a very important rôle in the various provinces of

* Mathematicians who busy themselves a great deal with the formal theory of four-dimensional space, seem to acquire a capacity for imagining this form as easily as the three-dimensional form with which we are all familiar. Therefore, despite the oft-repeated statements to the contrary, it is not impossible to imagine four-dimensional space. Only, we must not attempt to represent to ourselves four-dimensional space in three-dimensional space, especially not without a knowledge of its properties.

mathematics. The naturally ordered groups always represent a single instance out of these possibilities, the source of which always lies outside the group concept, that is, it proceeds from the things themselves which are united into a group.

25. Numbers. An especially important group in the linear order is that of the *integral numbers*. Its origin is as follows:

First we abstract the difference of the things found in the group, that is, we determine, although they are different, to disregard their differences. Then we begin with some member of the group and form it into a group by itself. It does not matter which member is chosen, since all are regarded as equivalent. Then another member is added, and the group thus obtained is again characterized as a special type. Then one more member is added, and the corresponding type formed, and so on. Experience teaches that never has a hindrance arisen to the formation of new types of this kind by the addition of a single member at a time, so that the operation of this peculiar group formation may be regarded as *unlimited* or *infinite*.

The groups or types thus obtained are called the *integral numbers*. From the description of the process it follows that every number has two neighbors, the one the number from which it arose by the addition of a member, and the other the number which arose from it by the addition of a member. In the case of the number one with which the series

begins, this characteristic is present in a peculiar form, the preceding group being *group zero*, that is, a group without content. This number in consequence reveals certain peculiarities into which we cannot enter here.

Now, according to a previous observation (p. 64), not only does the order bring every number into relation with the preceding one, but since this last for its part already possesses a great number of relations to all preceding, these relations exert their influence also upon the new relation. This fact gives rise to extraordinarily manifold relations between the various numbers and to manifold laws governing these relations. The elucidation of them forms the subject of an extensive science.

26. Arithmetic, Algebra, and the Theory of Numbers. From this regular form of the number series numerous special characteristics can be established. The investigations leading to the discovery of these characteristics are purely scientific, that is, they have no special technical aim. But they have the uncommonly great practical significance that they provide for all possible arrangements and divisions of numbered things, and so have instruments at hand ready for application to each special case as it arises. I have already pointed out that in this lies the positive importance of the theoretical sciences. For *practical* reasons the study of them must be as *general* as possible. This science is called *arithmetic*.

Arithmetic undergoes an important generalization if the individual numbers in a calculation are disregarded and *abstract signs* standing for any number at all are used in their place. At first glance this seems superfluous, since in every real numerical calculation the numbers must be reintroduced. The advantage lies in this, that in calculations of the same form the required steps are formally disposed of once for all, so that the numerical values need be introduced only at the conclusion and need not be calculated at each step. Moreover, the general laws of numerical combination appear much more clearly if the signs are kept, since the result is immediately seen to be composed of the participating members. Thus, *algebra,* that is, calculation with abstract or general quantities, has developed as an extensive and important field of general mathematics.

By the theory of numbers we understand the most general part of arithmetic which treats of the properties of the "numerical bodies" formed in some regular way.

27. Co-ordination. So far our discussion has confined itself to the *individual* groups and to the properties which each one of them exhibits *by itself.* We shall now investigate the relations which exist *between two or more groups,* both with regard to their several members and to their aggregate.

If at first we have two groups the members of which are all differentiated from one another, then

any one member of the one group can be co-ordinated with any one member of the other group. This means that we determine that the same should be done with every member of the second group as is done with the corresponding member of the first group. That such a rule may be carried out we must be able to do with the members of all the groups whatever we do with the members of one group. In other words, no properties peculiar to individual members may be utilized, but only the properties that each member possesses as a member of a group. As we have seen, these are the properties of *association*.

First, the co-ordination is *mutual*, that is, it is immaterial to which of the two groups the processes are applied. The relation of the two groups is reciprocal or symmetrical.

Further, the process of co-ordination can be extended to a third and a fourth group and so on, with the result that what has been done in one of the co-ordinated groups must happen in all. If hereby the third group is co-ordinated with the second, the effects are quite the same as if it were co-ordinated directly with the first instead of indirectly through the second. And the same is true for the fourth and the fifth groups, etc. Thus, co-ordination can be extended to any number of groups we please, and each single group proves to be co-ordinated with every other.

Finally, a group can be co-ordinated with itself,

each of its members corresponding to a certain definite other member. It is not impossible that individual members should correspond to themselves, in which case the group has *double members,* or *double points.* The limit-case is *identity,* in which every member corresponds to *itself.* This last case cannot supply any special knowledge in itself, but may be applied profitably to throw light on those observations for which it represents the extreme possibility.

28. Comparison. If we have two groups A and B, and if we co-ordinate their members severally, three cases may arise. Either group A is exhausted while there are members remaining in B, or B is exhausted before A, or, finally, both groups allow of a mutual co-ordination of *all* their members. In the first case A is called, in the broader sense of the word, *smaller* than B, in the second B is called smaller than A, in the third the two groups are said to be of *equal magnitude.* The expression, "B is greater than A," is equivalent to the expression, "A is smaller than B," and inversely.

It is to be noted that the relations mentioned above are true, whether the members are considered as individually different from one another or whether the difference of the members is disregarded, and they are treated as alike. This comes from the fact that every definite co-ordination of a group can be translated into every other possible co-ordination by exchanging two members at a time in

pairs. Since in this process one member is each time substituted for another, and a gap therefore can never occur in its place, the group in the new arrangement can be co-ordinated with the other group as successfully as in the old arrangement. At the same time we learn from this that in every co-ordination of a group with itself, independently of the arrangement of its members, it must prove equal to itself.

By carrying out the co-ordination proof is further supplied of the following propositions:

$$\text{If group A is} \left\{ \begin{array}{l} \text{greater than} \\ \text{equal to} \\ \text{smaller than} \end{array} \right\} \text{group B}$$

$$\text{and group B is} \left\{ \begin{array}{l} \text{greater than} \\ \text{equal to} \\ \text{smaller than} \end{array} \right\} \text{group C}$$

$$\text{then group A is also} \left\{ \begin{array}{l} \text{greater than} \\ \text{equal to} \\ \text{smaller than} \end{array} \right\} \text{group C}$$

From this it follows that any collection of finite groups whatsoever, of which no one is equal to the other, can always be so arranged that the series should begin with the smallest and end with the greatest, and that a larger should always follow a smaller. *This order would be unequivocal,* that is,

there is only one series of the given groups which has this peculiarity. As we shall soon see, the series of integers is the purest type of a series so arranged.

In comparing two infinitely large groups by co-ordination, it may be said on the one hand that never will one group be exhausted while the other still contains members. Accordingly, it is possible to designate two unlimited or infinite groups (or as many such groups as we please) as *equal* to each other. On the other hand, the statement that in both groups each member of the one is co-ordinated with a member of the other has no definite meaning on account of the infinitely large number of members. *The definition of equality is therefore not completely fulfilled*, and we must not loosely apply a principle valid for finite groups to infinite groups. This consideration, which may assume very different forms according to circumstances, explains the "paradoxes of the infinite," that is, the contradictions which arise when concepts of a definite content are applied to cases possessing in part a different content. If we wish to attempt such an application, we must in each instance make a special investigation as to the manner in which the relations on their part change by the change of those contents (or premises). As a general rule we must expect that the former relations will not remain valid in these circumstances without any change at all.

In the course of these observations we have learned how co-ordination can be used for obtaining a number of fundamental and multifariously applied principles. From this alone the great importance of co-ordination is evident, and later we shall see that its significance is even more far-reaching. *The entire methodology of all the sciences is based upon the most manifold and many-sided application of the process of co-ordination,* and we shall have occasion to make use of it repeatedly. Its significance may be briefly characterized by stating that it is the most general means of bringing connection into the aggregate of our experiences.

29. **Counting.** The group of integral numbers, because of its fundamental simplicity and regularity, is by far the best basis of co-ordination. For while arithmetic and the theory of numbers give us a most thorough acquaintance with the peculiarities of this group, we secure by the process of co-ordination the right to presuppose these peculiarities and the possibility of finding them again in every other group which we have co-ordinated with the numerical group. The carrying out of such co-ordination is called *counting,* and from the premises made it follows *that we can count all things in so far as we disregard their differences.*

We count when we co-ordinate in turn one member of a group after another with the members of the number series that succeed one another until

the group to be counted is exhausted. The last number required for the co-ordination is called the *sum* of the members of the counted group. Since the number series continues indefinitely, every given group can be counted.

Numerals have been co-ordinated with *names* as well as with *signs*. The former are different in the different languages, the latter are international, that is, they have the same form in all languages. From this proceeds the remarkable fact that the written numbers are understood by all educated men, while the spoken numbers are intelligible only within the various languages.

The purpose of counting is extremely manifold. Its most frequent and most important application lies in the fact that the amount affords a *measure for the effectiveness or the value* of the corresponding group, both increasing and decreasing simultaneously. A further number serves as a basis for divisions and arrangements of all kinds to be carried out within the group, whereby liberal use is made of the principle that everything that can be effected in the given number group can also be effected in the co-ordinated counted group.

30. Signs and Names. The co-ordination of names and signs with numbers calls for a few general remarks on co-ordination of this nature.

The possibility of carrying out the formal operations effected in one of the groups upon the co-ordinated group itself facilitates to an extraordinary

extent the practical shaping of the reality for definite purposes. If by counting we have ascertained that a group of people numbers sixty, we can infer without actually executing the steps that it is possible to form these men in six rows of ten, or in five rows of twelve, or in four rows of fifteen, but that we cannot obtain complete rows if we try to arrange them in sevens or elevens. These and numberless other peculiarities we can learn of the group of men from its amount, that is, from its co-ordination with the numerical group of sixty. In co-ordination, therefore, we have a means of acquainting ourselves with facts without having to deal directly with the corresponding realities.

It is clear that men will very soon notice and avail themselves of so enormous an advantage for the mastery and shaping of life. Thus, we see the process of co-ordination in general use among the most primitive men. Even the higher animals know how to utilize co-ordination consciously. When the dog learns to answer to his name, when the horse responds to the "Whoa" and the "Gee" of his driver there is in each case a co-ordination of a definite action or series of actions, that is, of a concept with a sign, or, in other words, of a concept with a member of another group; and in this there need not be the least similarity between the things co-ordinated with each other. The only requirement is that on the one hand the co-ordinated sign should be easily and definitely expressed and be

to the point, and that, on the other hand, it should be easily "understood," that is, *comprehended* by the senses and unmistakably *differentiated* from other signs co-ordinated with other things.

Thus, we find that the most frequent concepts of co-ordinated sound signs form the beginnings of *language* in the narrower sense. It is very difficult to ascertain for what reasons the particular forms of sound signs have been chosen, nor is it a matter of great importance. In the course of time the original causes have disappeared from our consciousness and the present connection is purely external. This is evident from the enormous difference of languages in which hundreds of different signs are employed for the same concept.

Now it would be quite possible to solve the problem of co-ordinating with each group of concepts a corresponding group of sounds, so that each concept should have its own sound, or, in other words, that the *co-ordination should be unambiguous.* It would not by any means be beyond human power to accomplish this, if it were not for the fact that the concepts themselves are still in so chaotic a state as they are at present. We have seen that the attempts of Leibnitz and Locke to draw up a system of concepts, if only in broad outline, have undergone no further development since. Even the most regulated concepts as well as the familiar concepts of daily life are in ceaseless flux, while the co-ordinated signs are comparatively more stable. But they,

too, undergo a slow change, as the history of languages shows, and in accordance with quite different laws from those which govern the change of concepts. The consequence is that in language the coordination of concepts and words is far from being unambiguous. The science of language designates the presence of several names for the same concept and of several concepts for the same name by the words synonym and homonym. These forms, which have arisen accidentally, signify so many *fundamental defects* of language, since they destroy the *principle of unambiguity* upon which language is based. In consequence of the false conception of its nature we have until now positively shrunk from consciously developing language in such a way that it should more and more approach the ideal of unambiguity. Such an ideal is in fact scarcely known, much less recognized.

31. The Written Language. Sound signs, to be sure, possess the advantage of being produced easily and without any apparatus, and of being communicable over a not inconsiderable distance. But they suffer under the disadvantage of transitoriness. They suffice for the purpose of temporary understanding and are constantly being used for that. If, on the other hand, it is necessary to make communications over greater distances or longer periods of time, sound signs must be replaced by more permanent forms.

For this we turn to another sense, the sense of

sight. Since optic signs can travel much greater distances than sound signs without becoming indistinguishable, we first have the optical telegraphs, which find application, though rather limited application, in very varying forms, the most efficient being the heliotrope. The other sort of optic signs is much more generally used. These are objectively put on appropriate solid bodies, and last and are understood as long as the object in question lasts. Such signs form the *written language* in the widest sense, and here, too, it is a question of co-ordinating signs and concepts.

What I have said concerning the very imperfect state of our present concept system is true also of these two groups. On the other hand, the written signs are not subject to such great change as the sound signs, because the sound signs must be produced anew each time, whereas the written signs inscribed on the right material may survive hundreds, even thousands of years. Hence it is that the written languages are, upon the whole, much better developed than the spoken languages. In fact, there are isolated instances in which it may be said that the ideal has well-nigh been reached.

As we have already pointed out, such a case is furnished by the *written signs* of numbers. By a systematic manipulation of the ten signs 0 1 2 3 4 5 6 7 8 9 it is not only possible to co-ordinate a written sign with any number whatsoever, but this co-ordination is strictly unambiguous, that is, each

number can be written in only one way, and each numerical sign has only one numerical significance. This has been obtained in the following manner:

First, a special sign is co-ordinated to each of the group of numbers from zero to nine. The same signs are co-ordinated with the next group, ten to nineteen, containing as many numbers as the first. To distinguish the second from the first group, the sign one is used as a prefix. The third group is marked by the prefixed sign two, and so on, until we reach group nine. The following group, in accordance with the principle adopted, has as its prefix the sign ten, which contains two digits. All the succeeding numbers are indicated accordingly. From this the following result is assured: First, no number in its sequence escapes designation; second, never is an aggregate sign used for two or more different numbers. Both these circumstances suffice to secure unambiguity of co-ordination.

It is known that the system of rotation just described is by no means the only possible one. But of all systems hitherto tried it is the simplest and most logical, so that it has never had a serious rival, and the clumsy notation with which the Greeks and Romans had to plague themselves in their day was immediately crowded out, never to return again upon the introduction of the Indo-Arabic notation, which has made its way in the same form among all the civilized nations and constitutes a uniform part of all their written languages.

The comparison of the spoken and the written languages offers a very illuminating proof of the much greater imperfection of the language of *words.* The number 18654 is expressed in the English language by eighteen thousand six hundred and fifty-four, that is, the second figure is named first, then the first, the third, the fourth, and the fifth. In addition, four different designations are used to indicate the place of the figures, -teen, -thousand, -hundred, and -ty. A more aimless confusion can scarcely be conceived. It would be much clearer to name the figures simply in their sequence, as one-eight-six-five-four. Besides, this would be unambiguous. If we should desire to indicate the *place value* in advance, we could do so in some conventional way, for example, by stating the number of digits in advance. This, however, would be superfluous, and ordinarily should be omitted.*

32. Pasigraphy and Sound Writing. There are two possibilities for co-ordination between concepts and written signs. Either the co-ordination is *direct,* so that it is only a matter of providing every concept with a corresponding sign, or it is indirect,

* The usual designation of the larger groups, ten, hundred, thousand, million, billion, etc., is also quite irrational. If it is our object to secure expressions for place values in as few words as possible, we find that the numbers of the form 10^{2n}, in which n is a whole number, must receive their own names, that is, 10, 100, 10,000, 100,000,000 etc. In this way the problem of designating as many numbers as possible by as few words as possible is solved.

the signs serving only the purpose of expressing the *language sound.* In the latter case the written language is based entirely upon the sound language, and the only problem, comparatively easy to solve, is to construct *an unambiguous co-ordination between sound and sign.* The Chinese script follows the direct process, but all the scripts of the European-American civilized peoples are based on the indirect process.

This, it is true, is the case only in ordinary, non-scientific language, while for science the European nations also have to a large extent built up a direct concept writing. One example of this we have seen in the number signs. Musical notation furnishes another instance, though by far not so perfect. The use of the different keys destroys the unambiguous connection between the pitch and the note sign, and the signatures placed at the beginning of a whole staff have the defect of removing the sign from the place where it is applied. Despite this imperfection musical notation is quite international, and every one who understands European music also understands its signs.*

Fundamentally we need not hesitate to recognize in *concept writing* or *pasigraphy* a more complete solution of the problem of sign arrangement. Even the very incomplete Chinese pasigraphy ren-

*It is not difficult to perfect musical notation with a view to unambiguity, a thing which would greatly facilitate the study of music.

ders possible written intercourse, especially for mercantile purposes, between the various East-Asiatic peoples who speak some dozens of different languages. But each language community translates the common signs into its own words, just as we do in the case of the number signs. But in order that such a system of representation should be complete it must fulfil a whole series of conditions for which scarcely a remote possibility is to be discerned at present.

At first the concepts could simply be taken as found in the words and grammatical forms of the various languages, and each one provided with an arbitrary sign. Such approximately is the Chinese system. But a system of that sort entails an extreme burdening of the memory, which results both from the great number of words and from the necessity of keeping the signs within certain bounds of simplicity. If we consider that the complex concepts are formed according to laws, to a large extent still unknown, from a relatively small number of *elementary* concepts, we may attempt to build up the signs of the complex concepts by the combination of those of the elementary concepts according to corresponding rules. Then it would only be necessary to learn the signs for the elementary concepts and the rules of combination in order for us to be able to represent all the possible concepts. This would provide even for the natural enlargement of the concept world, since every new

elementary concept would receive its sign and would then serve as the basis from which to deduce all the complex concepts dependent upon it. In fact, even should a concept hitherto regarded as elementary prove to be complex, it would not be difficult to declare that its sign, like the name of an extinct race, is dead, and after the lapse of sufficient time to use it for other purposes.

The numerical signs offer an excellent example for the elucidation of this subject, and at the same time serve as a proof that in limited provinces the ideal has already been attained. Another very instructive example is furnished by the chemical formulas, which, though they use the letters of the European languages, do not associate with them sound concepts, but chemical concepts. Since the chemical concepts are co-ordinated with certain letters, it is possible, in the first place, to denote the composition of all combinations qualitatively by the combination of the corresponding letters. But since quantitative composition proceeds according to definite relations which are determined by a variety of specific numbers peculiar to each element and called its combining weight, we need only add to the sign of the element the concept of the combining weight in order to represent in the second place the quantitative composition. Further, the multiples mentioned can also be given. Since, moreover, there are various substances which, despite equal composition, possess different properties, the

attempt has been made to express this new manifoldness by the position of the element signs on the paper, and in more recent times also by space representation. And here, too, rules have been worked out in which the scheme affords a close approach to experience. This example shows how, by the constant increase of the complexity of a concept (here the chemical composition), ever greater and more manifold demands are made upon the co-ordinated scheme. The form of expression first chosen is not always adequate to keep pace with the progress of science. In this case it must be radically changed and formed anew to meet the new demands.

33. **Sound Writing.** In point of unambiguity of co-ordination *phonetic writing* is far more imperfect than concept writing. It is obvious that in phonetic writing all the faults already present in the co-ordination between concept and sound are transferred to the written language. To these are added the defects as regards unambiguity occurring in co-ordination between sound and sign from which no language is free. In some languages, in fact, notably in English, these defects amount to a crying calamity. The principle of unambiguity would require that there should never be a doubt as to the way in which a spoken word is written, and as little doubt as to the way in which a written word is spoken. It needs no proof to show how often the principle is violated in every language. In the German language the same sound is represented by f, v,

and ph; in the English by f and ph. And in both German and English quite different sounds are associated with c, g, s, and other letters. *The fact that orthographic mistakes can be made in the writing of any language is direct proof of its imperfection,* and the oftener this possibility occurs the more imperfect is the language in this respect. We know that the spelling reforms begun in Germany more than ten years ago and recently in America and England, have for their object unambiguity in the co-ordination between sign and sound. Still it must be admitted that this tendency has not always been pursued undeviatingly. A few innovations, in fact, undoubtedly represent a step backward.

34. The Science of Language. A comparison of our investigations—which we cannot present in detail but only indicate—with the science of language or philology as taught in the universities and in a great number of books, reveals a great difference between them. This academic philology makes a most exhaustive study of relations, which from the point of view of the purpose of language are of no consequence whatever, such as most of the rules and usages of grammar. A study of this sort must naturally confine itself to a mere determination of whether certain individuals or groups of individuals have or have not conformed to these rules. Even the chief subject of modern comparative philology, the study of the relations of the word forms to one another and their changes in the course of history,

both within the language communities and when transferred to other localities, appear to be quite useless from the point of view of the theory of co-ordination. For it is indeed of little moment to us to learn by what process of change, as a rule utterly superficial, a certain word has come to be co-ordinated with a concept entirely different from the one with which it had been previously co-ordinated. Of incomparably greater importance would be investigations concerning the gradual change of the concepts themselves, although by no means as important as the real study of concepts. To be sure, such investigations are much more difficult than the study of word forms set down in writing.

Nevertheless, on account of a historical process, which it would lead us too far afield to discuss, an idea of such word investigations has been formed which is wholly disproportionate to their importance. And if we ask ourselves what part such labors have taken in the progress of human civilization, we are at a loss for an answer. Students of the *science* of language make a sharp distinction between it and the *knowledge* of language, which is regarded as incomparably lower. But while a knowledge of language is important in at least one respect, in that it presents to us the cultural material set down in other languages, or makes them accessible in translation to those who do not know foreign languages, philology is of no service in this respect at all, and the pursuit of it will seem

as inconceivably futile to future science as the scholasticism of the middle ages seems to us now.

The unwarranted importance attached to the historical study of language forms is paralleled by the equally unwarranted importance ascribed to grammatical and orthographic correctness in the use of language. This perverse pedantry has been carried to such lengths that it is considered almost dishonorable for any one to violate the usual forms of his mother tongue, or even of a foreign language, like the French. We forget that neither Shakespeare nor Luther nor Goethe spoke or wrote a "correct" English or German, and we forget that it cannot be the object of a true cultivation of language to *preserve* as accurately as possible existing linguistic usage, with its imperfections, amounting at times to absurdities. Its real object lies rather in the appropriate *development* and *improvement* of the language. We have already mentioned the fact that in one department, orthography, the true conception of the nature of language and of its development is gradually beginning to assert itself. Among most nations efforts are being made to improve orthography with a view to unambiguity, and when once sufficient clearness is had as to the object aimed for in spelling, there will be no special difficulty in finding the required means to attain it.

But in all the other departments of language we are still almost wholly without a conception of the genuine needs. Though the example of the Eng-

lish language proves that we can entirely dispense with the manifold co-ordinations in the same sentence as appearing in the special plural forms of the adjective, verb, pronoun, etc., yet the idea of consciously applying to other languages the natural process of improvement unconsciously evolved in the English language seems not to have occurred even to the boldest language reformers. So strongly are we all under the domination of the "schoolmaster" ideal, that is to say, the ideal of preserving every linguistic absurdity and impracticability simply because it is "good usage."

A twofold advantage will have been attained by the introduction of a *universal auxiliary language* (p. 183). Recently the efforts in that direction have made considerable progress. In the first place it will provide a general means of communication in all matters of common human interest, especially the sciences. This will mean a saving of energy scarcely to be estimated. In the second place, the superstitious awe of language and our treatment of it will give way to a more approprate evaluation of its technical aim. And when by the help of the artificial auxiliary language, we shall be able to convince ourselves daily how much simpler and completer such a language can be made than are the "natural" languages, then the need will irresistibly assert itself to have these languages also participate in its advantages. The consequences of such progress to human intellectual work in general would be ex-

traordinarily great. For it may be asserted that philosophy, the most general of all the sciences, has hitherto made such extremely limited progress only *because it was compelled to make use of the medium of general language.* This is made obvious by the fact that the science most closely related to it, mathematics, has made the greatest progress of all, but that this progress began only after it had procured both in the Indo-Arabic numerals and in the algebraic signs a language which actually realizes very approximately the ideal of unambiguous coordination between concept and sign.

35. Continuity. Up to this point our discussions have been based on the general concept of the *thing,* that is, of the individual experience differentiated from other experiences. Here the fact of *being different,* which, as a general experience, led to the corresponding elementary concept, appeared in the foreground in accordance with its generality. But in addition to it there is another general fact of experience, which has led to just as general a concept. It is the concept of *continuity.*

When, for example, we watch the diminution of light in our room as it grows dark in the evening, we can by no means say that we find it darker at the present moment than a moment before. We require a perceptibly long time to be able to say with certainty that it is now darker than before, and throughout the whole time *we have never felt the increase* of darkness from moment to moment, al-

though theoretically we are absolutely convinced that this is the correct conception of the process.

This peculiar experience, our failure to perceive individual parts of a change, the reality of which we realize when the difference reaches a certain degree, is very general, and, like memory, is based upon a fundamental physiological fact. It has already been noted by *Herbart*, but its significance was first recognized by *Fechner*, and has since then become generally known in physiology and psychology under the name of *threshold*. *Next to memory the threshold determines the fundamental lines of our psychic life.*

The threshold therefore means that whatever state we are in *a certain finite amount of difference or change must be stepped over* before we can perceive the difference or change. This peculiarity appears in all our states or experiences. We have already given an example for the phenomena of light and darkness. The same is true of differences in color and of our judgments as to tone pitch and tone strength. Even the transition from feeling well to feeling ill is usually imperceptible, and it is only when the change occurs in a very brief time that we become conscious of it.

The physical causes of these psychic phenomena need be indicated only in brief. In all our experiences an existing chemico-physical state in our sense organs and in the central organ undergoes a change. Now experiments with physical apparatus

have shown that such a process always requires a finite, though sometimes a very small, quantity of work, or, generally speaking, energy, before it can be brought about at all. Even the finest scale, sensitive to a millionth of a gram, remains stationary when only a tenth of a millionth is placed upon it, although we can *see* a body of such minute weight under the microscope. In the same way it requires a definite expenditure of energy in order to bring the sense organs, or the central organ, into action, and all stimuli less than this limit or threshold produce no experience of their presence.

By this the difficult concept of continuity is evoked in our experience. The transition from the light of day to the darkness of evening proceeds *continuously,* that is, at no point of the whole transition do we notice that the state just passed is different from the present one, while the difference over a wider extent of the experience is unmistakable. If we wish to bring vividly to our minds the contradiction to other habits of thought which this involves, we need only to represent to ourselves the following instance. I will compare the thing A at a certain time with the thing B, which is so constructed that though objectively different from A, the difference has not yet reached the threshold. From experience, therefore, I must take A to be equal to B. Then I compare B with a thing C, which is objectively different from B in the same way as A is from B, though here, too, the difference is still

within the threshold, though very near it. I shall also have to take B as equal to C. But now if I compare A directly with C, the sum of the two differences oversteps the threshold value, and I find that A is different from C. This, then, is a contradiction of the fundamental principle that if A=B and B=C, A=C. This principle is valid for *counted* things, which, in consequence, are discontinuous, but not for continuous things susceptible by our senses. If in spite of this it is applied to continuous things or *magnitudes* in the narrower sense, we must bear in mind that it is just as much a case of an *extrapolation to the non-existing ideal instance* (p. 46) as in the case of the other general principles, which, though they are derived from experience, nevertheless, for practical purposes, transcend experience in their use.

The examples cited above prove also that these relations are by no means confined to the judgments we derive on the basis of immediate sensations. When by means of the scale we compare three weights, the differences of which lie within the limit of its sensitiveness but approach closely to it, we can arrive in a purely empirical and objective way also at the contradiction A=B, B=C, but A≠C. In weight and measurement, therefore, we hold fast to the principle that the relations cited have no claim to validity outside the limit of their possible errors. Accordingly, though the non-equation of A≠C can be observed, the difference of both values

cannot be greater than at utmost the sum of the two threshold values.

These considerations also give us a means of appraising the oft-repeated statement that in contradistinction to the physical laws the mathematical laws are absolutely accurate. The mathematical laws do not refer to real things, but to imaginary ideal limit cases. Consequently they cannot be tested by experience at all, and the demands science makes on them lie in quite a different sphere. Their nature must be such that *experience should approximate them infinitely,* if certain definite well-known postulates are to be more and more fulfilled, and that the various abstractions and idealizations should be so chosen as not to contradict one another. Such contradictions have by no means always been avoided. But we must not regard them as inherent in the inner organization of our mind, as Kant did. These contradictions spring from careless handling of the concept technique, by which postulates elsewhere rejected are treated as valid. We have already come across an instance of such relations in the application of the concept of equality to unlimited groups (p. 84).

We must be guided by the same rules of precaution in answering the question whether the things felt as continuous—for example, space and time—are "truly" continuous, or whether in the last analysis they must not be conceived of as discontinuous. The various sense organs, and still more, the various

physical apparatus with which we examine given states, are of very varying degrees of "sensibility," that is, the threshold for distinguishing the differences may be of very different magnitudes. Therefore, a thing which is discontinuous for a sensitive apparatus will behave as if it were continuous with a less sensitive apparatus. Accordingly, we shall find so many the more things continuous the less sharply developed our ability is to differentiate.

While this circumstance makes it possible that we should regard discontinuous things as continuous, time relations in certain circumstances produce the opposite effect. Even if in a process the change is continuous but very rapid, and the new state remains unchanged for a certain time, we easily conceive of this sequence as discontinuous. We cannot resist this view of the process when the change occurs in a shorter time than the threshold time of our mind for each step in the process. But since this threshold changes with our general condition, one and the same process can appear to us both continuous and discontinuous according to circumstances. Here, therefore, we have a cause through the operation of which, with advancing knowledge, more and more things will become recognized as *continuous*.

Now if we turn to *experience*, we find, as the sum total of our knowledge, that for the sake of expediency we approach everything with the presumption that it is *continuous*. This aggregate ex-

perience finds its expression in such sayings as "Nature makes no jumps," and similar proverbial generalizations. But we must emphasize the fact once more that in deciding matters in this way we deal solely with questions of expediency, not with questions of the nature of our mental capacity.

36. **Measurement.** Measuring is in a certain way the opposite of counting. While, in counting, the things are regarded in advance as *individual,* and the group, therefore, is a body compounded of discontinuous elements, measuring, on the other hand, consists in *co-ordinating numbers with continuous things,* that is, in applying to continuous things a concept formed upon the hypothesis of discontinuity.

It lies in the nature of such a problem that the difficulty of adaptation must crop out somewhere in the course of its attempted solution. This is actually shown by the fact that measurement proves to be an unconcluded and inconcludable operation. If, in spite of this, measurement may and must justly be denoted as one of the most important advances in human thought, it follows that those fundamental difficulties can practically be rendered harmless.

Let us picture to ourselves some process of measurement—for example, the determination of the length of a strip of paper. We place a rule divided into millimeters (or some other unit) on the strip, and then we determine the unit-mark at which the strip ends. It turns out that the strip does not end exactly at a unit-mark, but *between* two unit-marks.

And even if the rule is provided with divisions ten or a hundred times finer, the case remains the same. In most cases a microscopic examination will show that the end of the strip does not coincide with a division. All that can be said, therefore, is that the length must lie *between n and n + 1 units,* and even if a definite number is given, the scientifically trained person will supplement this number by the sign $\pm f$, in which f denotes the possible errors, that is, the limit within which the given number may be false.

We see at once how the characteristic concept of threshold, which has led to the conception of the continuous, immediately asserts itself when in connection with discontinuous numbers. The adaptation of the threshold to numbers can be carried as far as it is possible to reduce the threshold, but the latter can never be made to disappear entirely.

The significance of measurement therefore lies in the fact that it applies the operation of counting with all its advantages (see p. 85) to *continuous* things, which as such do not at first lend themselves to enumeration. By the application of the unit measure a discontinuity is at first artificially established through dividing the thing into pieces, each piece equal to the unit, or imagining it to be so divided. Then we count the pieces. When a quantity of liquid is *measured* with a liter this general process is carried out physically. In all other less direct methods of measurement the physical process is sub-

stituted by an easier process equally good. Thus, in the example of the strip of paper we need not cut it up into pieces a millimeter in length. The divided rule is available for comparing the length of any number of millimeters that happen to come under consideration, and we need only read off from the figures on the rule the quantity of millimeters equal to the length of the strip, in order to infer that the strip can be cut up into an equal number of pieces each a millimeter in length.

After it has been made possible to count continuous things in this way, the numeration of them can then be subjected to all the mathematical operations first developed only for discrete, directly countable things. When we reflect that our knowledge of things has given them to us *preponderatingly as continuous,* we at once see what an important step forward has been made through the invention of measurement in the intellectual domination of our experience.

37. **The Function.** The concept of continuity makes possible the development of another concept of greater universality, which can be characterized as an extension of the concept of causation (p. 31). The latter is an expression of the experience, if A is, B is also, in which A is understood to be a definite thing at first conceived of as immutable. Now it may happen that A is not immutable, but represents a concept with continuously changing characteristics. Then, as a rule, B will also be of

that nature, so that *every special value or state of B corresponds to every special value or state of A.*

Here, in place of the reciprocal relation of two definite things, we have the reciprocal relation of two more or less extended groups of similar things. If these things are continuous, as is assumed here (and which is extremely often the case), both groups or series, even though they are finite, contain an endless quantity of individual cases. Such a relation between two variable things is called a function. Although this concept is used chiefly for the reciprocal relation of *continuous* things, there is nothing to hinder its application to discrete things, and accordingly we distinguish between continuous and discontinuous functions.

The intellectual progress involved in the conception of the reciprocal relation of entire *series* or groups to one another, as distinguished from the conception of the relations between *individual* things, is of the utmost importance and in the most expressive manner characterizes the difference between modern scientific thought and ancient thought. Ancient geometry, for example, knew only the cases of the acute, right, and obtuse angled triangle, and treated them separately, while the modern geometrician represents the side of the triangle as starting from the angle zero and traversing the entire field of possible angles. Accordingly, unlike his colleague of old, he does not ask for the particular principles bearing upon these particular

cases, but he asks in what continuous relation do the sides and angles stand to one another, and he lets the particular cases develop from out of one another. In this way he attains a much profounder and more effectual insight into the whole of the existing relations.

It is in mathematics in especial that the introduction of the concept of continuity and of the function concept arising from it has exercised an extraordinarily deep influence. The so-called *Higher Analysis,* or *Infinitesimal Analysis,* was the first result of this radical advance, and the *Theory of Functions*, in the most general sense, was the later result. This progress rests on the fact that the magnitudes appearing in the mathematical formulas were no longer regarded as certain definite values (or values to be arbitrarily determined), but as *variable,* that is, values which may range through all possible quantities. If we represent the relation between two things by the formula B=f(A), expressed in spoken language by B *is a function* of A, then in the old conception A and B are each individual things, while in the modern conception A and B represent an inexhaustible series of possibilities embracing every conceivable individual case that may be co-ordinated with a corresponding case.

Herein lies the essential advantage of the concept of continuity. It is true that it also introduces into calculation the above-mentioned contradictions

which crop up in the ever-recurring discussions concerning the infinitely great and the infinitely small. The system introduced by Leibnitz of calculating with *differentials,* that is, with infinitely small quantities, which in most relations, however, still preserve the character of finite quantities from which they are considered to have been derived, has proved to be as fruitful of practical results as it is difficult of intellectual mastery. We can best conceive of these differentials as the expression of the law of the threshold, which law gave rise to, or made possible, the relation between the continuous and the discrete.

38. The Application of the Functional Relation. I have already shown (p. 34) how the first formulation of a causal relation which experience yields can be purified and elaborated by the multiplication of the experience. The method described was based upon the fact that the necessary and adequate factors of the result were obtained by eliminating successively from the "cause" the various factors of which its concept was or could be compounded, and by concluding from the result, that is, the presence or absence of the "effect," as to the necessity or superfluity of each factor.

Obviously the application of this process presupposes the possibility of eliminating each factor in turn. Very often it is not possible, and then in place of the inadequate method of the individual case the *method of the continuous functional rela-*

tion steps in with its infinitely greater effectiveness. If in most cases we cannot *eliminate* the factors one by one, there are very few instances in which it is not possible to *change* them, or to observe the result in the automatically changed values of the factors. But then we have the principle that for the causal relation *all such factors are essential the change of which involves a change of the result.*

It is clear that this signifies a generalization of the former and more limited method. For the elimination of the factor means that its value is reduced to zero. But now it is no longer necessary to go to this extreme limit; it suffices merely to influence in some way the factor to be investigated.

It is true that here the difference in the result cannot be expressed with a "yes" or a "no," as before. It can only be said that it has changed *partly*, more or less. From this it can be seen that the application of this process requires more refined methods of observation, especially for measuring, that is, for determining values or magnitudes. On the other hand, we must recognize how much deeper we can penetrate into the knowledge of things by the application of the measuring process. Each advance in precision of measurement signifies the discovery of a new stratum of scientific truth previously inaccessible.

39. The Law of Continuity. From the fact that natural phenomena in general proceed continuously we can deduce a number of important and generally

applicable conclusions which are constantly used for the development of science.

When a relation of two continuously varying values of the form $A=f(B)$ is conjectured, we convince ourselves of its truth by observing for different values of A the corresponding values of B, or reversely. If we find that changes in the one correspond to changes in the other, the existence of such a relation is proved, at first only for the observed values, though we never hesitate to conclude that for the values of A lying between the observed values, but themselves not yet observed, the corresponding values of B will also lie between the observed values. For example, if the temperature at a given place has been observed at intervals of two hours, we assume without hesitancy that in the hours between when no observations were made, the values lie between the observed values. If we indicate the time in the usual manner by horizontal lines and the temperature for the general periods of time by longitudinal lines, the law of continuity asserts that all these temperature points lie in a steady line, so that when a number of points lying sufficiently near one another is known, the points between can be derived from the steady line which may be drawn through the known points. This very commonly applied process will yield the more accurate results the nearer the known points are to one another, and the simpler the line.

The application of the law of continuity or steadi-

ness, therefore, means no less than that it is possible, from a finite, frequently not even a very large, number of individual results, to obtain the means of predicting the result for an infinitely large number of unexamined cases. The instrument derived from this law, therefore, is an eminently *scientific* one.

The value of this instrument is still greater if it succeeds in expressing the relation A=f(B) in strict mathematical form. First, the result of the determination of a number of individual values of that function is represented as a table of co-ordinated values. By the graphic process above described, or by its equivalent, the mathematical process of interpolation, this table is so extended that it also supplies all the intermediate values. But this is still a case of a mechanical co-ordination of the corresponding values. Often we succeed, especially in the relation of simple or pure concepts, in finding a general mathematical rule by which the magnitude A can be derived from the magnitude B, and reversely. This is the only instance in which we speak of a natural law in the quantitative sense.

Thus, for example, we can observe what volume a given quantity of air occupies when successively subjected to different pressures. If we arrange all these values together in a table, we can also calculate the volume for all the intermediate pressures. But on close inspection of the cor-

responding numbers of pressure and volume we notice that they are in inverse ratio, or that when multiplied by one another their products will be the same. If we denote the space by v and the pressure by p, this fact assumes the mathematical form $p. v=K$, in which K is a definite number depending upon the quantity of air, the unit of pressure, etc., but remaining unchanged in an experimental series in which these factors stay the same. The general functional equation $A=f\ (B)$ becomes the definite $p=\frac{K}{v}$. And this formula enables us to determine by a simple calculation the volume for any degree of pressure, provided the value of K has been once ascertained by experiment.

At first we have a right to such a calculation only within the province in which the experiments have been made, and the simple mathematical expression of the natural law has for the time being no further significance than that of a specially convenient rule for interpolation. But such a form immediately evokes a question which demands an experimental answer. How far can the form be extended? That there must be a limit is to be directly inferred from the consideration of the formula itself. For if we let $p=o$, then $v=$infinity, both of which lie beyond the field of possible experience.

Similar considerations obtain in all such mathematically formulated natural laws, and each time,

therefore, we must ask what the *range of validity* of such an expression is, and answer the question by experiment.

While in this discussion the mathematically formulated natural law seems to have the nature only of a convenient formula of interpolation, we are nevertheless in the habit of regarding the discovery of such a formula as a great intellectual accomplishment, which so impresses us that we frequently call it by the name of the discoverer. Now, wherein lies the more significant value of such formulations?

It lies in the fact that simple formulas are discovered only *when the conceptual analysis of the phenomenon has advanced far enough.* The very simplicity of the formula shows that the concept formation which is at the basis of it is especially serviceable. In Ptolemy's theory of the motion of the planets the means for calculating their positions in advance was given just as in the theory of Copernicus. But Ptolemy's theory was based on the assumption that the earth stands still, and that the sun and the other planets move. The assumption that the sun stands still and that the earth and the other planets move greatly facilitates the calculation of the position of the planets. In this lay the primary value of the advance made by Copernicus. It was not until much later that it was found that a number of other actual relations could be represented much more fittingly by means of the same hypoth-

esis, and thus the Copernican theory has come to be generally recognized and applied.

The significance of the law of continuity and its field of application have by no means been exhausted by what has been said above. But later we shall have a number of occasions to point out its application in special instances, and so cause its use to become a steady mental habit with the beginner in scientific research.

40. Time and Space. Time and space are two very general concepts, though without doubt not elementary concepts. For besides the elementary concept of continuity which both contain, time has the further character of being one-seried or one-dimensional, of not admitting of the possibility of return to a past point of time (absence of double points) and of absolute onesidedness, that is, of the fundamental difference between before and after. This last quality is the very one not found in the space concept, which is in every sense symmetrical. On the other hand, owing to the three dimensions it has a *three*fold manifoldness.

That despite this radical distinction in the properties of space and time all of our experiences can be expressed or represented within the concepts of space and time, is very clear proof that experience is much more limited than the formal manifoldness of the conceivable. In this sense space and time can be conceived as natural laws which may be applied to all our experiences. Here at the same

time the subjective-human element of the natural law becomes very clear.

The properties of time are of so simple and obvious a nature that there is no special science of time. What we need to know about it appears as part of physics, especially of mechanics. Nevertheless time plays an essential rôle in *phoronomy*, a subject which we shall consider presently. In phoronomy, however, time appears only in its simplest form as a one-seried continuous manifoldness.

As for space, the presence of the three dimensions conditions a great manifoldness of possible relations, and hence the existence of a very extensive science of bodies in space, of *geometry*. Geometry is divided into various parts depending upon whether or not the concept of measurement enters. When dealing with purely spacial relations apart from the concept of measurement it is called geometry of position. In order to introduce the element of measurement a certain hypothesis is necessary which is undemonstrable, and therefore appears to be arbitrary and can be justified only because it is the simplest of all possible hypotheses. This hypothesis takes for granted that a rigid body can be moved in all directions in space without changing in measure. Or, to state the inverse of this hypothesis, in space those parts are called equal which a rigid body occupies, no matter how it is moved about.

We are not conscious of the extreme arbitrariness

of this assumption simply because we have become accustomed to it in school. But if we reflect that in daily experience the space occupied by a rigid body, say a stick, seems to the eye to undergo radical changes as it shifts its position in space and that we can maintain that hypothesis only by declaring these changes to be "apparent," we recognize the arbitrariness which really resides in that assumption. We could represent all the relations just as well if we were to assume that those changes are real, and that they are successively undone when we restore the stick to its former relation to our eye. But though such a conception is fundamentally practicable in so far as it deals merely with the space picture of the stick, we nevertheless find that it would lead to such extreme complications with regard to other relations (for example, the fact that the weight of the stick is not affected by the change of the optic picture) that we do better if we adhere to the usual assumption that the optical changes are merely apparent.

In this connection we learn what an enormous influence the various parts of experience exert upon one another in the development of science. In every special generalization of experiences, that is, in every individual scientific theory, our aim is not only to generalize this special group of experiences in themselves, but at the same time to join such other experiences to them as expedience demands. If the effect of this necessity is on the one hand to render

the elaboration of an appropriate theory more difficult, it has on the other hand the great advantage of affording a choice among several theories of apparently like value, and thus making possible a more precise notion of the reality. For example, for the understanding of the mutual movements of the sun and the earth it is the same whether we assume that the sun moves about the earth or the earth about the sun. It is not until we try to represent theoretically the position of the other planets that we see the economic advantage of the second conception, and facts like Foucault's experiment with a pendulum can be represented only according to this second conception in our present state of knowledge.

Likewise, the assumption on which scientific geometry goes, that space has the same properties in all directions, conflicts with immediate experience. In immediate experience we make a sharp distinction between below and above, although we are prepared to admit the "homogeneity" of space in the horizontal direction. This is due, as physics teaches, to the fact that we are placed in a field of gravitation which acts only from above downward and which permits free horizontal turnings, although it imparts a characteristic difference to the third direction. Since considerations of another kind enable us to place ourselves in a position in which we ignore this field of gravitation in the investigation of space, geometry abstracts this element and

disregards the corresponding manifoldness. In the theory of the gravitation potential, on the other hand, this very manifoldness is made the subject of scientific investigation.

The common application of the concepts of space and time results in the concept of *motion*, the science of which is called phoronomics. In order to make this new variable subject to measurement we must arrive at an agreement or convention as to the way in which to measure time. For since past time can never be reproduced we actually experience only unextended moments, and have no means of recognizing or defining the equality of two periods of time by placing them side by side, as we can in the case of spacial magnitudes. We help ourselves by saying *that in uninfluenced motions equal periods of time must correspond to the equal changes in space.* We regard the rotation of the earth on its axis and its revolution about the sun as such uninfluenced motions. The two depend upon dissimilar conditions, and the empirical fact that the relation of the two motions, or the relation between the day and the year, remains practically the same, sustains that assumption, and at the same time shows the expediency of the given definition of time.

Analytic geometry, the application of algebra to geometric relations, occupies a noteworthy position, from the point of view of method, in the science of space. It yields geometric results by means of cal-

culation, that is, by the application of the *algebraic* material of symbols we can obtain data concerning unknown *spacial* relations. An explanation is necessary of how by a method apparently so extraneous such results as these can be attained.

The answer lies again in the general principle of co-ordination, which in this very case receives a particularly cogent illustration. Three algebraic signs, x, y, and z, are co-ordinated with the three variable dimensions of space. First, the same independent and constant variability is ascribed to these signs, and, further, the same mutual relations are assumed to subsist between them as actually exist between the three-spacial dimensions. In other words, precisely the same kind of manifoldness is imparted to these algebraic signs as the spacial dimensions possess to which they are co-ordinated, and we may therefore expect that all the conclusions arising from these assumptions will find their corresponding parts in the spacial manifoldness. Accordingly, a co-ordinated spacial relation corresponds to every change of those algebraic formulas resulting from calculation, and if such changes lead to an algebraically simple form, then the spacial form corresponding to it must show an analogous simplicity. Here, therefore, we have a case such as was described under simpler conditions on p. 86 of operations undertaken with one group and repeated correspondingly in the co-ordinated group. And it is only the great difference

in the things of which in this case the two groups are composed—spacial relations on the one side and algebraic signs on the other—that creates the impression of astonishment which was felt very strongly at the invention of this method, and which is still felt by students with talent for mathematics when they first become acquainted with analytical geometry.

41. Recapitulation. Before we proceed to consider the fundamentals of other sciences, it is well to make a general résumé of the field so far traversed. Since the later sciences, as we have already observed, make use of the entire apparatus of the earlier sciences, the mastery of them must be assured in order to render their special application possible.

This does not mean that one must have complete command of the entire range of those earlier sciences in order to pursue a later one. Mere human limitations would prevent the fulfilment of such a demand. As a matter of fact, successful work can be done in one of the later sciences even if only the most general features of the earlier ones have been clearly grasped. Nevertheless, the rapidity and certainty of the results are very considerably increased by a more thorough knowledge of the earlier sciences, and the investigator, accordingly, should seek a middle road between the danger of insufficient preparation for his special science and the danger of never getting to it from sheer preparation. In any circumstances he must be prepared al-

ways, even though it be in later age, to acquire those fundamental aids so soon as he feels the need of them for carrying out any special work. It is generally acceded that without logic the adequate pursuit of science is impossible. Nevertheless, the opinion is widely current, even among men of science, that everybody has command of the needful logic without having studied it. No more than a man can learn of himself to use the calculus, even if he may have discovered unaided some of its elementary principles, can he acquire certainty and readiness in the use of the logical rules generally necessary, unless he has made the necessary studies. It is true that the scientific works of the great pioneers and leaders in the special sciences furnish practical examples of such logical activity. But complete freedom and security are acquired only on the basis of conscious knowledge.

We have now seen how, from the physiological construction of our mental apparatus, the process of concept formation and the experience of concept connections are the basis of the whole of mental life. The laws of the mutual interaction of the most general or elementary concepts operated in the formation of the concepts, *thing, group, co-ordination.* Here were found the fundamentals of logic or the science of concepts. A special process of abstraction yielded the concept of *number,* and with it the corresponding field of *mathematics,* arithmetic, algebra, and the theory of numbers.

By means of the second fundamental fact of physiology, the *threshold,* another elementary fact was explained, that of *continuity.* The co-ordination of individual things under the influence of this concept was expanded into the *co-ordination of continuous phenomena-series,* and yielded the correspondingly more general concept of the *function.* From the application of the number concept to continuous things, the idea of *measurement* resulted. In mathematics the concept of continuity led to higher *analysis* and the *theory of functions.* Finally, the concept of continuity proved to be an inexhaustible aid for the extension of scientific knowledge and for the formulation of natural laws in mathematical form.

PART III

THE PHYSICAL SCIENCES

42. General. In the formal sciences we began the specialization of the object from the most general concept of thing conceivable, possessing no other characteristic attribute than its capability of being distinguished from other things; and we carried the specialization so far that we could follow in its movements an object definite as to time and space. This object, to be sure, was defined only in that it occupied a definite space, and accordingly had a definite form. As a matter of fact, the spacial thing of geometry and phoronomy reveals no further attributes.

It is here that the physical sciences enter into their dominion one after the other, and fill the empty space of the geometric thing with definite attributes. These are the secondary qualities of Locke, of which he assumed that they do not belong so much to the bodies themselves as that they merely appear to us so on account of the nature of our human sense organs. Now that our knowledge concerning the nature of those properties as well as the structure of our sense organs is much

more thorough, we have more definite ideas also of the subjective part of the corresponding experiences, and in a large measure are able to separate it from the objective part.

All properties which physical bodies in contradistinction to geometric bodies possess can be traced back to a fundamental concept, which, in conjunction with the concepts explained in the former chapter, serves to characterize and distinguish the physical structure. For example, the fact that we can distinguish cubes of equal size but of different material, different temperature, and different luminosity, can be traced back always and entirely to the different kinds of energy acting in the geometric space in question. The concept of energy, therefore, plays approximately the same rôle in the physical sciences as the concept of thing in the formal sciences, and the essentials of this new field of science are the comprehensive knowledge and development of this concept. Because of its great importance it has long been known and applied in individual forms. But the systematization of the physical sciences relative to energy is a matter of only recent date.

43. Mechanics. Recently many scientists have taken exception to the traditional division of mechanics into *statics*, or the science of equilibrium, and *dynamics*, or the science of motion, because it does not correspond to the essence of the thing, equilibrium being only the limit-case of motion.

However, the classic presentations of this science are based on that division, so that it must express an essential difference. This difference we can clearly recognize through the application of the concept of energy to mechanics. We then learn that statics is the science of work, or the energy of position, and that dynamics is the science of living force, or of the energy of motion.

By *work* in the mechanical sense we mean the expenditure of force required for the locomotion of physical bodies. While a cube of lead is geometrically equal to a cube of glass, we experience a great difference between them when we lift them from the floor to a table. We call the cube of lead heavier than the glass cube, and we find it requires more work to raise the former than the latter. For psychologic reasons this judgment becomes especially clear when the work required to lift the lead cube marks the limit of our physical capacity.

Work depends not only upon the difference described above, but also upon the distance through which it is exerted. It increases in proportion as the distance increases. In mechanics work is proportional both to the distance and to that peculiar property which in the given example we call *weight*. But a more general concept has been formed for that property in the mechanical sense, called *force,* of which weight constitutes but a special instance. Whenever there is a resistance combined with a change of place we speak of a force, *and the*

product of the force and the distance we call work.

The cause of this kind of concept formation is the following: There are a great number of different machines, all of them possessing the peculiarity that work can be put into them at a definite place and taken out at another place. Now, centuries of experience have shown that it is impossible to obtain more work from such mechanical machines than has been put into them. As a matter of fact, the work obtained is always less than the work put in, and the two approach equality as the machine approaches perfection. It is to such ideal machines, therefore, that *the law of the conservation of work* applies. This law states that, though a given quantity of work may be changed in the most manifold ways as to direction, force, etc., it is impossible to change its *quantity*.

The reason we can judge of this fact with such certainty is because for many centuries a number of the ablest mechanicians have sought for a solution of the problem of perpetual motion, that is, for the construction of a machine from which more work can be gotten than is put into it. All such attempts have failed. But the positive result secured from these apparently futile efforts is the law of the conservation of work. The greatness and importance of this result will become apparent in the further course of our study.

Here for the first time we meet with a law ex-

pressing the *quantitative* conservation of a thing, which may none the less undergo the most varied qualitative changes. With the knowledge of this fact we involuntarily combine the notion that it is the "same" thing that passes through all these transformations, and that it only changes its outward form without being changed in its essence. Such ideas, it is true, are widespread, but they have a very doubtful side to them, since they correspond to no distinct concept. If we want to call the quantitative magnitude of the product of the force and distance the "essence" of work, and the determination of the force and the distance according to magnitude and direction, which come under consideration for each special value, as its "form," then, of course, there is no objection to be made to mere nomenclature. But we must bear in mind that the difference obtaining here lies exclusively in the fact that the amount of work measured quantitatively remains unchanged, while its factors undergo simultaneous and opposite changes.

This discovery, that there is a magnitude which can be quantitatively determined, and which, as experience shows, remains unchanged, however much its factors may change, invariably results not only in a very simple and clear formulation of the corresponding natural law, but also corresponds to the general tendency of the human mind to work out conceptually "the permanent in change." If, in accordance with the word-sense, we denote everything

which persists under changing conditions by the name of *substance, we encounter in work the first substance* of which we have attained knowledge in our scientific journeys. In the history of the evolution of human thought this substance has been preceded by others, especially by the weight and mass of ponderable bodies (which are also subject to a law of conservation), so that at present we are inclined to connect with the word substance a tacit secondary sense of ponderability. But this is a remnant of the still very widely spread mechanistic theory of the universe, which, though it has almost finished its rôle in physics, will presumably continue to persist for a long time to come in the popularly scientific consciousness in accordance with the laws of collective thought.

44. Kinetic Energy. The law of the conservation of work is by no means true of all cases in which work is expended or converted, but, as has been said, only of *ideal* machines, that is, of such cases which do not exist in reality. But while in imperfect machines there is at least an approximation to this law, there are besides countless normal cases in which we cannot even speak of an approximation. When, for example, a stone falls to the ground from a certain height, a certain quantity of work is expended, which is equal to that by means of which the stone can be raised again to its original height. This quantity of work apparently disappears entirely when the stone remains lying on the

ground. We shall discuss this case later. Or the falling of the stone can be so guided that it can lift itself again. This happens, for instance, when, by fastening the stone to a thread, it is forced to move in a curved path, or to perform pendular oscillations. In that case, it is true, the stone will fall to the lowest point which the thread permits, and so will there have lost its work without having done any other work in the meantime. But it has entered a condition by virtue of which it raises itself again, so that (as before, only in the ideal limit-case) it reaches its former height, and so has lost no work. For this moment, too, then, the law of the conservation of work obtains. But in the meantime new relations have arisen.

What distinguishes the stone moving like a pendulum from the stone which simply falls is, that at its lowest point it has not remained lying still, but possesses a certain velocity. By means of this it lifts itself again, and after it has reached its former height, it has lost its velocity. *Therefore, there is a reciprocal relation between the work which it loses and the velocity which it gains,* and the question may therefore be put, How can this relation be represented mathematically? Experience teaches that in every such case a function of the velocity and of another property of the body, called *mass,* can be established in such a way that this function, called the *kinetic energy* of the body, increases precisely as much as the amount of work the body has ex-

pended, and *vice versa*. The sum of the kinetic energy of the body and of the *work* is therefore *constant*, and the clearest mode of conceiving of this relation is by assuming *that work can be transformed into kinetic energy and vice versa* in such a way that given amounts of the two magnitudes are equal or equivalent to one another. Naturally, this is only an abbreviated way of expressing the actual relations, for it might just as well be assumed that the work really disappears and the kinetic energy really originates anew, and that the disappearance of the one substance only happens regularly to coincide with the origin of the other. But it is this regular conjunction of phenomena that constitutes the sole ground of every *causal* relation, and in such a sense we are justified *in regarding the disappearing work as the cause of the kinetic energy that arises*, and to designate this relation summarily as a transformation.

By the inclusion of cases in which work is converted into kinetic energy the law of the conservation of work therefore becomes *the law of the conservation of the sum of work and kinetic energy*. We are thereby compelled to extend the concept of substance, which at first contains only work, to the sum of both magnitudes, and to introduce a new name for this enlarged concept.

It will soon appear that all cases of imperfect machines, in which work disappears without giving rise to an equivalent amount of kinetic energy, can,

with a corresponding enlargement of the concept, be likewise included in the law of conservation. For experience has shown that in such cases something else arises, heat, light, or electric force, etc. This generalized concept, which embraces all natural processes and permits the sum of all corresponding values to be expressed by a law of conservation, we call *energy*. The law in question, therefore, is:

In all processes the sum of the existing energies remains unchanged.

The principle of the conservation of work in perfect machines proves to be an ideal special instance of this general law. A perfect machine is one in which work changes into nothing but *work* of another kind, and not into a different kind of energy. Then each side of the equation which expresses the general law of energy, namely,

Energy that has disappeared = energy that has arisen,

contains only the magnitude of the work, and expresses the law of the conservation of work. If, on the other hand, as in the case of the pendulum, the work increasingly changes part by part into kinetic energy, and *vice versa,* the equation during the first period is:

Work that has disappeared = kinetic energy that has arisen,

and during the second period in which the pendulum rises again,

Kinetic energy that has disappeared = work that has arisen.

Thus, while work can be called a substance only in a limited sense, since its conservation is limited only to perfect machines, we may call energy a substance unqualifiedly, since in every instance of which we know the principle has been maintained *that a quantity of any energy never disappears unless an equivalent quantity of another energy arises.* Accordingly, this law of the conservation of energy must be taken as a fundamental law of the physical sciences. But not only do all the phenomena of physics, including chemistry, occur within the limits of the law of conservation, but until the contrary is proved the law of conservation must also be regarded as operative in all the later sciences, that is, in all the activities of organisms, so that all the phenomena of life must also take place within the limits of the law of conservation. This corresponds to the general fact, which I have emphasized a number of times, that all the laws of a former science find application in all the following sciences, since the latter can only contain concepts which by specialization, that is, by the addition of further characteristics, have sprung from the concepts of the former or more general sciences.

45. **Mass and Matter.** It has been noted above that kinetic energy depends upon another magnitude beside velocity. A conception of its nature can be obtained when we try to put different bodies in mo-

tion. In doing so the muscles of the arm perform certain quantities of work, and we feel whether the quantities are greater or smaller. In this way we obtain a clear consciousness of the fact that different bodies require quite different quantities of work for the same velocity. The property which comes into play here is called *mass,* and mass is proportional to the work which the various bodies require to attain the same velocity. Since the work and the velocity can be measured very accurately by appropriate means, mass also lends itself to a correspondingly accurate measurement.

All known ponderable bodies have mass. That means there is a regular connection between the property which makes a body tend to the earth with a certain definite force (called weight) and the property by virtue of which a body assumes certain velocities under the influences of motive causes. We can readily conceive that it is possible for us to learn only of such bodies as are heavy, that is, bodies which are *held* by the earth, since the others, if they exist at all, would naturally have left the earth long ago. That all these bodies also have mass is to be explained in a similar way. For a body of mass zero would at each impulse assume infinitely great velocity, and could therefore never be the object of our observation. Consequently, by reason of the physical conditions obtaining on the earth's surface, the bodies known to us must combine both properties, mass and weight.

The name given to this concept of the combined presence of mass and weight in space is *matter*. Experience shows that there is a law of *conservation* for these magnitudes also, according to which *whatever changes we may produce in bodies possessing weight and mass, no change will occur in the sum of their weight and mass.* According to the nomenclature previously introduced we must therefore call weight and mass substances, since they remain the same as to quantity, no matter what changes they may undergo. However, it is usual to apply the name substance to the concept of matter composed of mass and weight. In fact, scientists often go so far as to limit the name to this single instance of the various laws of conservation, and to take substance to mean exclusively the combination of mass and weight. This is connected with the conception which we are about to discuss, that all natural phenomena can ultimately be conceived as the motion of matter. Through the greater part of the nineteenth century this conception, called *scientific materialism*, was accepted almost without opposition. At present it is being more and more recognized that it was only an unproved assumption, which the development of science daily proves to be more untenable.

46. Energetic Mechanics. In the light of our previous observations the branch of science traditionally known as mechanics appears as the science of work and of kinetic energy. Furthermore, statics

is shown to be the science of work, while dynamics, besides treating of kinetic energy in itself, also treats of the phenomena of the change of work into kinetic energy, and *vice versa*. We shall find the same relation again later, only in more manifold forms. Every branch of physics proves to be the science of a special kind of energy, and to the knowledge of each kind of energy must be added the knowledge of the relations by which it changes to the other forms of energy and *vice versa*. It is true that in the traditional division of physics this system has not been strictly carried out, since an additional and very influential motive for classification has been the regard paid to the various human sense organs.

Nevertheless this ground does not lie in the field of physics, but in that of physiology, and must, therefore, be abandoned in the interest of strict systematization.

Of the physical sciences mechanics was the first to develop in the course of historical evolution. A number of factors contributed to this end—the wide distribution of mechanical phenomena, their significance to human life, and the comparative simplicity of the principles of mechanics, which made it possible to discover them at an early date. Most to be noted is, that of all departments of physics mechanics is the first which lent itself to comprehensive *mathematical* treatment. It is true that the mathematical treatment of mechanics was possible only after idealizing assumptions had been made—

perfect machines and the like—so that the results of this mathematical treatment not infrequently had very little to do with reality. The mistake of losing sight of the physical problem and of making mechanics a chapter of mathematics has not always been avoided, and it is only in most recent times that the consciousness has again arisen that the classical mechanics, in arbitrarily limiting itself to extreme idealized cases, sometimes runs the risk of losing sight of the aim of science.

47. The Mechanistic Theories. Because the evolution of mechanics antedates that of the other branches of physics, mechanics has largely served as a model for the formal organization of the other physical sciences, just as geometry, which has been handed down to us from antiquity in the very elaborate form of Euclid, has largely been used as a model for scientific work in general. Such methods of analogy prove to be extremely useful at first because they serve as a guide to indicate when and where new sciences, in which all possibilities are open, can be got hold of. But later on such analogies are apt to be harmful. For each new science soon requires new methods, by reason of the peculiar manifoldness which it has to deal with, and the finding and the introduction of these new methods are easily delayed, and, as a matter of fact, often have been delayed, because scientists could not free themselves soon enough from the old analogy.

By its being based upon memory the human mind is so constructed that it cannot assimilate something entirely new. The new must in some way be connected with the known in order that it may be organically embodied in the aggregate of concepts. Therefore, it is the first involuntary impulse of our mind, in the presence of new experiences or thoughts, to look about for such points at which a linking of the unknown to the known seems possible. In the case of mechanics this necessity for finding connecting links has acted in such a way that the attempt has been made, and is still being made, to conceive and represent all physical phenomena as mechanical.

The impulse to this was first given by the extraordinary successes which mechanics has attained in the generalization and prediction of the *motions of the heavenly bodies*. The names of Copernicus, Kepler, and Newton mark the individual steps in the mechanization of astronomy. The cause of this lies in the fact that the heavenly bodies actually approximate very closely the ideal of the purely mechanical form with which classical mechanics deals. These successes encourage the attempt to apply these mental instruments that were productive of such rich results to all other natural phenomena. An old theory, according to which all physical things are composed of the most minute solid particles of matter called *atoms,* supported these tendencies and invited the attempt to regard the little world of atoms as subject to the same laws as had been found

to apply so successfully to the great world of the stars.

Thus we see how this mechanistic hypothesis, the assumption that all natural phenomena can be reduced to mechanical phenomena, comes as if it were a self-understood matter, and with its claim to be a profound interpretation of nature it scarcely permits the question as to its justification to be raised at all. And the effects here have been the same as I described above in cases in which inferences from analogy are accepted too extensively or too credulously. While it is true, no doubt, that the mechanical hypothesis at first was fruitful of results in special research, because it facilitated the putting of the question—for example, we need think only of the atomic hypothesis in chemistry—later, the efforts to find further hypothetic help for the inadequacies of the hypothesis that gradually came to light, have not infrequently led scientific research to pseudo-problems, that is, to questions which are questions only in hypothesis, but to which no actual reality can be shown to correspond. Such problems, therefore, are by their very nature *insoluble,* and constitute an inexhaustible source of differences of scientific opinion.

The most flagrant of the injurious consequences of the mechanistic hypothesis appear in the scientific treatment of the mental phenomena. Ready as scientists were to represent all other life phenomena, such as digestion, assimilation, and even generation

and propagation, as the consequence of an extremely complicated play of certain atoms, their courage never went so far as to apply this principle to mental life and to consider that by mechanics the last word had been said on the subject.

It is because of this hesitancy to bring mental phenomena under the same mechanistic principle as all the other phenomena that the philosophical systems had to search for some other means to connect the mental world with the mechanical, and the efforts of the philosophers to bring about this end have been most varied. Of the various doctrines that have come down to us, that of the *pre-established harmony* proposed by Leibnitz is in the ascendant in our day, and is now called the theory of the *psycho-physical parallelism*. According to this theory it is assumed that the mental world exists alongside, and quite independent of, the mechanical, but that the things have been so prearranged that mental processes take place simultaneously with certain mechanical processes (according to some, with all mechanical processes) in such a way that, although the two series do not influence each other in the least, they always correspond to each other precisely. How such a relation has come about and how it is maintained remains unsaid, or is left to future explanation.

We need only think of the content of this hypothesis with an unbiased mind to lose all relish for it at once. In fact, it has no other *raison d'être* than

the presumption that the mental and the mechanical world are opposed to each other. As soon as we abandon the thesis that the non-mental world is exclusively mechanical, we acquire the possibility again of finding for the theory of mental phenomena a constant and regular connection with the theories of all other phenomena, especially with the phenomena of life. Therefore it will be found most expedient in every respect, instead of rendering scientific research one-sided and almost blind to nonconforming facts by preconceived hypotheses, such as the mechanistic hypothesis, to seek, as hitherto, from step to step, the new elements of manifoldness which must be taken account of in the progressive upbuilding of science and to limit ourselves faithfully to them in the formation of general ideas.

48. Complementary Branches of Mechanics. The field of pure or classical mechanics is limited to the above two kinds of energy, work and kinetic energy, though these do not exhaust the manifoldness of the mechanical energies. Accordingly, other branches of mechanics dealing with the corresponding phenomena are added to the classical mechanics described above.

If by mechanical energies we understand all energies in which *changes of space are connected with changes of energy,* there are as many different forms as there are spacial concepts that seem applicable. *Form, Volume,* and *Surface* of bodies in space are especially recognizable as the field of

action for energy, which shows different properties or manifoldnesses according to each of these relations.

The *energy of form* is manifested in bodies (solid or rigid bodies) that maintain a definite shape because every change of shape is connected with work or with the expenditure of some other energy. If the changes are small, the bodies are of such a nature that they return to their former condition of their own accord after the force exerted upon them has ceased to act. This property is called *elasticity*. However, the theory of elasticity, which has been extensively and rationally developed, is regarded as belonging rather to mathematical physics in general than to mechanics in particular. In greater changes of shape the energy of form, or elastic energy, passes into other forms, and the body does not return to its former shape after the force has been removed.

Other bodies have no energy of form (or only in an infinitesimally slight degree), so that they allow of changes of form without the expenditure of work, but their volume can be changed only by work. These are divided into two classes. First, the *liquids,* which have a definite volume (corresponding to the definite shape of solids), the changes of which in *every* sense, both compression and expansion, require work. Secondly, the *gases* with volume energy in only one sense of the word, in which only the compression of volume requires

work, while in expansion a certain amount of work is thrown off. Such bodies can exist only so long as the expenditure of their volume energy by spontaneous expansion is prevented by the presence of a counter energy, as, for example, the elasticity of the walls of a vessel. This tendency is called *pressure*.

Finally, there are energy qualities at the surfaces between various kinds of bodies which come into play at the change of these surfaces. They always lie in such a direction that the enlargement of the surfaces requires work, and hence, by reason of the law of conservation of energy, cannot proceed by itself. In cases where there has been an inverse kind of energy present, that is, one which diminishes with increasing surface, it also has been active as a rule, thus bringing about the disappearance of the existing boundaries.

Since the seat of this kind of energy is in the surfaces (or superficies), it is called *surface-energy*. The phenomena depending upon it manifest themselves most clearly at the surface boundaries between *liquids* and *gases*. They are called *capillary phenomena*. This strange name, derived from the word *capilla,* hair, has its origin in the fact that because of surface-energy liquids rise in tubes which they wet, and the narrower the tube the higher they rise. If the lumen of the tube is as fine as a *hair,* a considerable rise can be observed. This is the entire connection between the name and the thing.

The mechanics of liquids is called *hydromechanics,* that of gases, *aeromechanics,* after the most familiar liquid, water, and the most familiar gas, air. The study of surface-energy under the name of the capillary theory forms part of theoretical physics. While formerly this branch, too, was regarded as a working part, or, rather, as a playing part, of mathematical problems, in more recent times extensive experimental research has made its entry in this province also, and has demonstrated the necessity of passing from the former abstractions or idealizations, which were carried altogether too far, to a better and profounder regard for the actually existing complexities.

49. The Theory of Heat. The various forms of energies the aggregate of which is comprehended in physics, have very different special characters. A systematic investigation has not yet been made of the characters of manifoldness by which, for example, work is distinguished from heat, electrical energy from kinetic energy, etc., nor of what are the essential properties peculiar to each individual energy. We feel certain that differences do exist, for otherwise the energies could not be distinguished, and we feel certain that these differences are very important, for doubt seldom arises as to the kind of energy to which a certain phenomenon is to be assigned. But just as we have no systematic table of the elementary concepts, so we are still without a systematic natural history of the forms of

energy in which the peculiarities of every species are characterized, and in which the entire material is so arranged according to these characteristics that we can take a general survey of it.

As regards heat energy, its foremost and most striking characteristic is its physiological effect. In our skin there are organs for the perception of heat as well as of cold, that is, for temperatures above and below the temperature of the skin. However, the temperature that these organs can bear without injury to themselves is of a very small range, beyond which physical apparatuses of all kinds must be used, such as "thermometers."

Heat is the simplest kind of energy from the point of view of manifoldness. Every heat quantity is marked by a temperature, just as a kinetic energy is marked by velocity. But while a velocity is determined in space so that velocities of equal magnitude have in addition a threefold infinite manifoldness in reference to direction, a temperature is characterized completely and unambiguously by a simple number, the degree of temperature. Two temperatures of equal degree can in no wise be distinguished, since temperature possesses no other possible manifoldness than degree.

The same property is found in heat energy itself. In heat energy we measure the quantity of energy itself and call it the *heat quantity,* while in some of the other kinds of energy, only the factors into which they can be divided are measured, and no

habitual conception of the energy itself is developed. A heat quantity is likewise fully indicated by its measure number.

That heat is an energy, that is, that it is developed in equal quantities from other kinds of energy, and can change back again into them, is a discovery which, despite its fundamental and general character, was not made before the forties of the nineteenth century. As often happens in cases of important scientific advances, the same idea came simultaneously to a number of investigators. The first to grasp and fully comprehend this idea was *Julius Robert Mayer* of Heilbronn, who published his results in 1842. Mayer not only showed that the imperfect machines (p. 134), which limit the validity of the law of the conservation of work, owe this peculiarity to the fact that they transform a part of the work into *heat,* and that when we take account of this part, the law of conservation holds perfectly good, but he also calculated, with extraordinary acumen, the mechanical equivalent of heat from the then existing data of physics. That is to say, he determined how many units of heat (in the measure then in use) correspond to a unit of work (in its specific measure) in the change from one to the other, and back. And this fundamental knowledge of the existence of a quantitatively unchangeable substance, arising from work, and capable of being transformed into it, Mayer did not limit in its application merely to heat. He was the first to con-

struct a table, which he made as complete as possible, of all the forms of energy then known, and to assert and prove the possibility of their reciprocal change into each other.

In view of this relation of the quantitative equivalent of the various forms of energy when transformed into one another, an attempt is being made at present to measure them all with the *same unit.* That is, some easily obtained quantity of energy is arbitrarily chosen as a unit and it is determined that in every other form of energy the unit shall be equal to the quantity obtained from that unit on its transformation into the energy in question. For formal reasons the kinetic energy of a mass of two grams which moves with the velocity of one centimeter in a second has been chosen as the unit. It is called *erg,* an abbreviation of energy. The amount is very small, and for technical reasons 10^{10} times greater unit is used. To raise the temperature of a gram of water one degree a quantity of energy equal to 41,830,000 ergs is required.

50. The Second Fundamental Principle. Another fundamental discovery has been made in connection with the heat form of energy, which, like the law of conservation, relates to all forms of energy, but has found its first and most important application in heat. While the law of conservation answers the question, how much of the new form of energy is developed if a given quantity of energy changes, but gives no clue as to when such

a change occurs, this second law asserts the condition under which such changes arise, and is therefore called the *second fundamental principle.*

The discovery of this law antedates *Mayer's* discovery of the law of conservation by about twenty years, and was made by a French military engineer, *Sadi Carnot,* who died soon afterward without having lived to see the recognition his great work obtained. *Carnot* asked himself the question, Upon what does the action of the steam engine, which had just then come into use, depend? This led him first to the more general question of the action of heat engines in general. He found that no heat engine could work unless the heat dropped from a higher to a lower temperature, just as no water wheel can work unless the water flows from a higher to a lower level, and he determined the conditions which an *ideal heat engine* must fulfil, that is, a machine in which the greatest possible value in work is obtained from heat. However, an ideal machine of this nature can be constructed in very different ways, and Carnot's discovery consists in the recognition of the fact *that the quantity of work obtained from the heat unit does not at all depend upon the peculiar construction of the ideal machine, but is determined solely by the temperature between which the heat transition takes place.* This follows from the following considerations:

In the first place an ideal engine must be *reversible,* that is, it must be capable of working both

ways, changing heat into work and work back into heat. Now, if we have two ideal engines between the same temperatures, and if we assume that engine A produces more work from the same quantity of heat than engine B, then let A move one way and let B move the other way with the work obtained from A. Since B produces less work from a given amount of heat, hence more heat from an equal amount of work, there will in the end be more heat at the higher temperature than was originally there. But experience teaches *that there is no means in nature by which heat in the absence of concomitant change could be caused to rise to a higher temperature.* Therefore an engine so constructed as to produce this result is impossible, And B cannot be of such a nature as to produce less work from the same quantity of heat than A.

The reverse is also impossible. For then we need merely couple the engines in the reverse way in order to obtain the same effect. Therefore, since B can do neither less nor more work than A, the two must do the same amount of work—which was to be proved.

It is obvious that this process of proof is similar to that by which the law of conservation was established. Because the arbitrary creation of energy from nothing is impossible there must be definite and immutable relations of change between the forms of energy. Because energy at rest does not spontaneously pass into conditions in which it can

do work, the efficiencies of the machines must have definite and unchangeable values. If, for example, we could cause heat of its own accord to rise to a higher temperature, we could also construct a perpetual motion machine which would always yield work at no expense. But this perpetual motion would not be one that creates work out of nothing, but one that extracts it from energy at rest. A perpetual motion machine of this nature, too, is, according to our experience, impossible, and this impossibility forms the content of the second fundamental principle.

On the face of it this apparently "self-evident" proposition does not reveal how fruitful of results it is when applied to the discovery of simple but not obvious relations. It can only be said here that the deductions from this principle form the chief content of the extensive science of thermodynamics, which deals with the changes of heat into other forms of energy. We must only emphasize the fact that the application of this law, as was already observed in stating it, is not confined to the changes of heat alone. It is a law rather which finds application in *all* the forms of energy. For in every form of energy there is a property which corresponds to temperature in heat, and upon the equality or the inequality of which depends whether the energy in question is at rest or ready for transformations. This property is called the *intensity* of the energy. In work, for instance, it is *force*,

in volume-energy it is *pressure*. If once the intensity in a body is equal, its energy is at rest, and it never again moves of its own accord.

Another form in which to present these relations is to make a distinction between *free* energy and energy *at rest*. If we have a heat quantity the temperature of which is higher than that of the surrounding objects, it can be used to do work only until its temperature has dropped to that of the surrounding objects. Although energy in abundance is still present, there is no longer any energy *capable of change*, or *free* energy. Since differences of temperature, like other differences of intensity, have a constant tendency to diminish, the amount of free energy on earth is constantly decreasing, and yet it is only this free energy that has value. For since all phenomena depend upon change of energy, and change of energy is possible only through free energy, *free energy is the condition of all phenomena*.

51. **Electricity and Magnetism.** While the knowledge of heat energy goes back to the most ancient periods of civilization, electrical and magnetic energies are relatively young acquisitions. The highly developed technical application of both with the rich harvests they have yielded belongs exclusively to most recent times.

Both these forms of energy, like those discussed above, are connected in the main with ponderable "matter," but in a much slighter and less regular

measure. While it is not possible as yet to render any given body free of heat (although lately the absolute zero point has been considerably approximated), freedom from electrical and magnetic energy is the normal condition of most bodies. This is connected with the peculiarity that electrical and magnetic properties are decidedly bi-symmetrical or *polar*. This property is not found in any other form of energy, and can serve as the special scientific characteristic of electricity and magnetism. This peculiarity shows itself in the concepts of positive and negative magnetism, and positive and negative electricity, and is due to the fact that two equal opposite quantities of electricity or magnetism, when added together, do not produce double their value, but nullify each other.*

The fact that electrical and magnetic energies generally exist only in a transitory state (with the notable exception of the magnetic condition of the earth) is probably the cause of our not having developed a sense organ for them, especially since their phenomena as they occur in nature have only

* For the sake of the layman it must be observed that those "quantities" are not energy magnitudes but factors of the electrical and magnetic energies. Energy itself in its various forms is an *exclusively positive magnitude*, and the result of the additions of their various amounts is always the sum, never the difference, of their numerical values. By the negative sign is understood the energy *expended* in contradistinction to the energy *received*. It is therefore nothing more than the indication of a mathematical operation.

occasionally and in very rare instances (thunderstorms) an influence upon us. On the other hand, the modern development of electrotechnics is based upon that property of electrical energy by virtue of which large quantities of it can be conducted along a thin wire over great distances without any considerable loss, and at the point desired can be easily changed into any other forms of energy. But since the collection and conservation of large quantities of electrical energy is hardly possible technically, the electrical apparatus must be so constructed that the quantities each time required should be produced at the moment they are used. The chief source of electricity is the chemical energy of coal, which is first transformed into heat, then into mechanical energy, and finally into electrical energy. This extremely roundabout process is necessary because a method technically practicable of transforming the chemical energy of coal directly into electrical energy has not yet been invented. On the other hand, mechanical energy can be easily and completely changed into electrical energy. Upon this is based the exploitation of much "water power," the energy of which could not be utilized but for the great capacity for change of the electrical form.

52. Light. The case of light in our day seems to be similar to that of sound, which, although it has its special sense organ in man, is yet no particular form of energy, but has been found to be a com-

bination of mechanical energies in an oscillatory or mutually changing state. It seems highly probable that light, too, is not a special form of energy, but a peculiar oscillatory combination of electrical and magnetic energies. It is true that the circle of proof is not yet quite closed, but the gaps have become so small that the above conclusion may at any rate be accepted as highly probable.

However that may be, light is an energy which, according to the known laws, travels through space with tremendous rapidity. We will call it *radiant energy,* since the part optically visible, to which alone the name light in its original sense belongs, represents an extremely small portion of a vast field, the properties of which change quite continuously from one end to the other.

Radiant energy is characterized as an oscillatory or wave-like process. So long as this fact was unknown (up to the beginning of the nineteenth century) it was thought that light consisted of minute spherical particles, which shot through space in a straight line with the tremendous velocity mentioned above. Later, in order to "explain" its wave nature, which in the meantime has come to be recognized, it was assumed to be due to the elastic vibrations of an all-pervading thing called *ether,* of which we know nothing else. This elastic undulatory theory has been abandoned in our time in favor of an *electromagnetic* theory supported by quite considerable experiential grounds. Whether it

will be spared the fate that has overtaken the older theories (or rather hypotheses) of light cannot as yet be predicted with any degree of certainty.

Radiant energy is of very marked importance in human relations. As light it serves, with the aid of the corresponding receiving organs, the eyes, as a more manifold means of intercommunication between our bodies and the outer world than any other form of energy. The energy quantities penetrating to us from the extreme limits of the world space mark the outermost limits of which we have knowledge in any way whatsoever, and finally the energy quantities radiating to us from the sun constitute the supply of free energy at the expense of which all organic life on earth is maintained. Even the chemical energy stored up in coal represents nothing else than accumulations of former sun radiation, which had been transformed by the plants into the permanent form of chemical energy.

Very recently other newly discovered forms of radiant energy have been added to light. They are produced in manifold circumstances, and some bodies emit them constantly. The scientific elaboration of these extremely manifold and unusual phenomena has not yet been carried so far that they can be reduced to a doubt-free system. But so much, it seems, is already apparent, that they are presumably not purely new forms of energy, but

rather very composite phenomena which may yield one or more new energies as component parts. But despite the peculiarity of these new rays, nothing certain has as yet been proved against the law of conservation itself.

53. Chemical Energy. Since chemical energy is only one of several forms of energy, there seems to be no justification for allotting it to a special science, since all the other forms of energy must be incorporated in physics.

But the actual existence of chemistry as a special science which has already many subdivisions is justified in the first place by the external fact that in practical life and in industry chemistry occupies a very wide field comparable, if not superior, to that of the whole of physics. In the next place, from the psychological point of view, it is found that the chemist's methods of reasoning and working are so different from those of the physicist that a division seems to be in order for that reason also. Finally, there is in the nature of chemical energy itself an important distinction which marks it off from the other forms.

While, for example, there is only one form of heat or of kinetic energy, and in electricity there are only the two forms of polar opposites, chemistry, even after the greatest theoretical reduction, possesses at least about eighty forms. That is, it possesses as many forms as there are *chemical elements*. The experiential law, that the elements cannot be

changed into one another,* also limits the corresponding changes of the chemical energies into one another, and thus characterizes the independence of these various forms. From this results a disproportionately greater manifoldness of relations, which find their expression in the many thousands of the individualized chemical substances or combinations.

This great manifoldness and the slight regularity hitherto found in connection with the properties and reciprocal relations of the numerous chemical elements renders modern chemistry more a descriptive than a rational science. It was no more than twenty years ago that an earnest and successful attempt was begun to apply the stricter methods of physics to the investigation of chemical phenomena. These labors, so far as they have gone, have yielded a great many far-reaching and comprehensive principles.

The significance of chemistry in human life is twofold. In the first place the energy of the human body, just as that of all other living organisms, depends chiefly upon the action of chemical energies in the most manifold forms. Of all the physical sciences, therefore, chemistry is the most important for biology, particularly for physiology. In the second place, as I have emphasized a number of times,

* Lately changes of elements into one another have been observed in individual instances, but in such peculiar circumstances that for the present we need not consider these discoveries, which have only just begun.

it possesses the peculiar property which enables it to be *preserved* for a long time without passing into other forms and being dissipated. Furthermore, energy in this form permits of the most powerful *concentration*. More of chemical energy can be stored in a given space than of any other form of energy. Both these properties, then, may be considered as the reason why organic beings are constituted chiefly by means of chemical energy. At any rate, it is due to these two peculiarities that chemical energy serves as the primary source for almost all the energy used in industry.

Further, the manifoldness of chemical energy is the cause of the peculiar manner in which it is transformed into other forms. In the other forms of energy the transformation can be effected by the body itself. Nothing else is required. If a stone is thrown and it hits against a wall, it loses its kinetic energy, the greater part of which changes into heat. But in order to liberate the *chemical* energy of, say, coal, the coal *alone* is not sufficient; *another* chemical substance is required, the oxygen of the air. The interaction of the two substances produces a new substance, and it is only during this process that a corresponding part of the chemical energy is liberated. There are a few chemical processes also (allotropic and isomeric changes) in which a single substance without the co-agency of another substance can give off energy. But the quantity of energy thus obtained is in-

finitely small as compared to that liberated by the interaction of two or more substances. Because of the necessity of two or more substances to co-operate in giving off chemical energy, the opportunity for the transformation of chemical energy is less than for the transformation of the other forms of energy, and this is the main reason why it can be conserved so long and so easily. All that is necessary is to prevent contact with another substance. This is a problem, it is true, which from the point of view of strict theoretical rigor it is almost impossible to solve. In practice, however, it can be easily solved for periods of time long enough at least to require special means to enable us to recognize that it is only a temporary and not a fundamental solution. Scientifically expressed, the cause of this is that the *diffusion* of the various substances in one another can theoretically never be completely eliminated, while on the other hand the velocity of the diffusion over distances measured only by decimeters is extremely low.

PART IV

THE BIOLOGIC SCIENCES

54. Life. Among the bodies in our environment that are ponderable and have mass the animate beings are so strikingly distinguished from the inanimate that in most cases we have not the slightest doubt whether a body belongs to the one kind or to the other, even if in some cases we happen not to be familiar with its peculiar form. In the first place, therefore, we must answer the question in a general way and tell what the distinguishing peculiarities are that mark them off one from the other.

The first peculiarity is this, that living organisms are not *stable* but *stationary* forms. This distinction is based upon the fact that a stable form is at rest or unchangeable in all its parts, while a stationary body, though it seems unchangeable in its form, internally undergoes a constant change of its parts. Thus, a brass faucet is a stable body, since it not only preserves its form and function permanently, but consists at all times of the same material and shows the same peculiarities, such as stains, defects in form, etc. It cannot be said, it is true, that it will remain completely unchanged for all

time. Its metal suffers a gradual chemical and mechanical deterioration. But this is not essential to the existence of the faucet, since the deterioration varies greatly with circumstances, and if conditions are ideal it can be reduced to zero.

On the other hand, the jet of water flowing from the faucet is a stationary body. In favorable circumstances it can assume a constant form, so that at a hasty glance it might be taken for a stable glass rod. On closer examination it will be found that the parts of water of which it is formed are not the same at any given instant as the instant before, each part that has flowed away being replaced by another just as large following it.

From this difference in the nature of the two bodies results a difference in their behavior. If I make a mark on the faucet with a file, the mark remains permanent. But even if I sever the entire water jet with a knife, the cut is healed the next moment, because by reason of the continuous flow of the water, the severed place is instantly eliminated from the body. Owing to this nature peculiar to stationary bodies, they have the capacity of *being healed* or of *regeneration*.

For a body to continue permanently in a stationary condition the material of which it is composed must be permanently *supplied*. If we turn off the faucet, the water jet immediately disappears or "dies." Evidently, therefore, a stationary body can subsist by its own means only if it has the property or

capacity to provide itself continually with the necessary material. This material consists in the main of ponderable or chemical substances of definite physical and chemical properties, and thus the *change of substance, metabolism,* appears as a necessary property of the stationary body. In order, however, that metabolism should take place we must have free *energy,* or energy having the capacity to work, since it is only free energy that can cause substances to change, just as every phenomenon in the world implies the equalization of free energy. For a stationary body to exist independently, therefore, it must have the property of being able spontaneously to possess itself of the necessary substances and of free energy. But since, as we have already said, the energy of organisms is stored up and used in the main in the form of chemical energy, the two tasks which a stationary body has to perform, that of meeting the need for substances and for energy, are as a rule externally combined. In organisms these two necessities combined are called *nutrition,* and thus we recognize in the capacity for *self-acquisition of nutrition* another essential property of organisms.

A third essential property of organisms is the capacity for *reproduction,* for the bringing forth of similar beings. It is never impossible that the balance between the receipts and expenditures of a stationary body should, in consequence of some external causes, be disturbed, even when under nor-

mal conditions it possesses the property of self-nutrition. If the disturbance remains below a certain point, then, as we have already stated, regeneration sets in. But the disturbance may rise above that point, in which case the body ceases to exist, or dies. Then a similar body will not arise unless the manifold necessities that have led to the origin of the first will combine again to produce the second. That such a thing is possible, that, in fact, it often happens, is shown, for example, by the waves of the ocean, which have a stationary character since, while they are composed of constantly changing masses of water, their form remains unchanged. The waves are destroyed in the breakers, but arise again and again through the action of the wind upon the surface of the water. But the more complex such bodies are the less easily they are formed, while once they have been formed and have found the conditions of their existence, their preservation is much easier.

Beings, therefore, which have the capacity to form similar bodies out of themselves regularly and at the right time can preserve their species much more easily than those in which this property is absent. Death has to a great extent lost its power over beings capable of reproduction. By way of illustration let us take another stationary thing, a flame. A flame is not an organism because it is not self-sustaining. Yet it multiplies itself. And while a single little flame soon dies out, the sea of

flame of a burning forest, which started from a single small flame, is well-nigh inextinguishable, and it cannot be fought in any other way than by letting it die its natural death and burn to the end.

Thus, while the fulfilment of the first two conditions, the stationary change and the self-supply of food, could produce bodies, which would be able to exist for a longer or shorter period, but which at some time would have to give way to other bodies of different form and nature, the capacity for reproduction creates the condition that forms of the *same species* continue to exist even after the existence of the individual has ceased.

These three properties constitute the essential characteristics of animate things or organisms.

That the organisms are all constructed upon the basis of chemical energy is a fact of experience which may be understood to imply that the other forms of energy are not capable of producing the above-mentioned conditions. This is due to the properties of chemical energy to which I have already called attention: its great concentration and, at the same time, its capacity for prolonged preservation. That chemical energy is the only form of energy suitable to life is obvious from the fact that in airship navigation, for example, the kinetic energy required for steering can be supplied only in the form of gasoline or hydrogen, that is, in the form of chemical energy, because any of the other forms would be much too heavy. The flight of a

bee or the swimming of a dolphin cannot be conceived of except as brought about through chemical energy.

That this chemical energy is essentially that of *carbon* has also been established by experience, although it is not quite universal, for the sulphur bacteria found their household upon the energy of sulphur. The cause of the preference of carbon is again to be sought in its special fitness for the purpose, due, on the one hand, to its wide distribution, and, on the other hand, to the exceeding manifoldness of its combinations.

Finally, the construction of the organisms from a peculiar combination of solid and liquid substances can be proved to be equally due to technical relations.

These three last-named peculiarities are therefore to be regarded as the special characteristics of the organisms with which we are acquainted on the surface of the earth in the conditions there prevailing. We need not regard them conceptually as unchangeable or irreplaceable. But the first three characteristics, namely, the stationary nature, self-supply of nutrition, and reproduction, we may regard as the *essential characteristics of organisms.* They constitute the frame within which everything must be found which we should recognize as living in the widest sense.

55. The Storehouse of Free Energy. If we ask whence the organisms obtain the free energy which

they require for the maintenance of their stationary existence, the answer is that *solar radiation* alone furnishes this supply. Without this permanent supply the free energies upon the earth, so far as our knowledge goes, would long ago have reached a state of equilibrium, and the earth's bodies would be stable, that is, dead and not stationary and living.

It is comprehensible, therefore, that machines should have evolved in the organism for *transforming the radiant energy of the sun into a permanent form*, and, as we have already learned, chemical energy is permanent, while radiant energy is an extremely transitory form of energy, that is, it changes very readily. The very fact that, owing to the change from day to night, the supply of radiant energy periodically ceases, makes the storing-up of energy for the night necessary to the existence of a form dependent upon it. Thus, we recognize in the *photochemical* processes, that is, in the transformation of radiant energy into chemical energy, the foundation of life on earth.

This work is done by the plants, which thus provide a store of free energy not only for their own needs but also for all the other organisms which possess themselves directly or indirectly of the plant-chemical supplies in order to utilize them for their individual purposes. In this manner nourishment in the widest sense is secured for all organisms, being based upon the regular supply of free

energy derived from the sun. This also explains the great chemical similarity of all organisms, which could not subsist if they were not so constructed as to be able to utilize the chemical energy in the form in which it is provided by the plants.

Of the great stream of free energy poured out from the sun into cosmic space the earth receives an extremely small portion (corresponding to the bit of space it occupies in the heavenly sphere as seen from the sun), and the plants collect and store up only a very small fraction of this portion received by the earth. Measurements have shown that in most favorable circumstances a plant leaf changes only about 1/50 of the radiant energy it receives into chemical energy. If we consider that only a small part of the surface of the earth is covered with plants and that during the winter no energy from the sun is stored up at all, we perceive what infinite possibilities for development there still are in arresting and storing up free energy. The part stored up by the plants flows from these into the countless streams, brooks, and veins of the other organisms, to end finally as used-up energy, or energy at rest. This energy is at rest, it is true, only in relation to the earth's surface. We do not know whether the radiation from the earth, which at present amounts to about as much as the radiation from the sun to the earth, is in its turn somewhere utilized.

While the free energy is poured out in such a

stream in one direction, the ponderable substances of which the organisms are made up *circulate* through plants and animals and back again. This is especially true of *carbon,* which is freed from its combination with oxygen, that is, from carbonic acid, by the sun energy transformed in the plants. While carbon serves to build up the plant body and represents its supply of chemical energy, the oxygen is returned to the air. These two substances are again chemically combined in the various organisms and the quantities of energy which were necessary for their decomposition are again available for the manifold functions of life. The product of the chemical combination, carbonic acid, returns to the air and is ready for renewed decomposition in the plants.

Thus, the entire mechanism of life can be compared to a water-wheel. The free energy corresponds to the water, which must flow in one direction through the wheel in order to provide it with the necessary amount of work. The chemical elements of the organisms correspond to the wheel, which constantly turns in a circle as it transfers the energy of the falling water to the individual parts of the machine.

56. The Soul. Our observations so far have shown the organisms to be extremely specialized individual instances of physico-chemical machines. Now we have to take into consideration a property which seems markedly to distinguish them from

the lifeless machines, and which we have already encountered in the very beginning of our treatise.

It is the property which we there called *memory*, and which we will define in a very general way as the quality by virtue of which the repetition in organisms of a process which has taken place a number of times is preferred to new processes, because it originates more easily and proceeds more smoothly. It is readily apparent that by this property the organisms are enabled to travel on the sea of physical possibilities as if equipped with a keel, by which the voyage is made stable and the keeping of the course is assured.

If we ask whether this is exclusively a quality of organisms the question cannot be answered affirmatively. Inanimate bodies also have something like the quality of adaptation. An accurate clock attains its valuable qualities only after it has been going for some time, and the best violin is "raw" until it has been "broken in." An accumulator must be "formed" before it can do its normal amount of work. All these processes are due to the fact that the repetition of the same process improves the action, that is, it facilitates or increases it.

Adaptation or memory, then, is not limited to organisms. In inanimate things, however, this property is comparatively rare. Memory, therefore, is to be regarded as another property of organisms representing an extreme specialization of the

inorganic possibilities. This is an important point of view for what follows.

In the first place, this property of adaptation facilitates and assures nourishment. If we take the fundamental idea developed by Darwin, that that predominates in the world which by virtue of its properties endures the longest time, then it is evident that a body which teleologically preserves and elaborates its nourishment will live longer than a similar body without this property. Moreover, by the general process of adaptation, these "teleological" properties come to be more greatly developed and more readily exercised in the body that lives longer, so that its long life gives it another advantage over its rival. Thus we can understand how this property of adaptation, which at first is to be conceived of as a purely physico-chemical quality is found developed in all organisms.

In its most primitive forms the quality of adaptation gives rise to the *phenomena of reaction,* or to *reflex* actions, that is, to a series of processes in the organism in response to the stimulus of an outward energy. This response is made in furtherance of the life of the organism. Reactions that serve a certain end, that is, teleological reactions, can naturally be developed to such stimuli alone to which the organism is frequently and regularly subjected. This is why adaptation to unusual phenomena is generally lacking, and in relation to them the organisms are often extremely unfit. The typical ex-

ample of this is the moth, which flies into the light and is burned.

As the reactions become more fixed they develop into longer and more complicated series, which then appear to us as *instinctive actions*. But here, too, we find the characteristic unsuitability when unwonted circumstances arise, even if the teleologic reactions to stimuli become more manifold.

Finally, there are the *conscious acts* which appear to us to be the highest degree of the series. It is with the teleologic regulation of these conscious acts, including the very highest activities of mankind, that this book deals. They are distinguished from instinctive action by the fact that they no longer proceed in a single and definite series, but are combined at need in the most manifold ways. But the fundamental fact, namely, that actions are based upon the repetition of coinciding experiences, at once appears here also, since the basis of the entire conscious life of the soul, the formation of *concepts,* is made possible only through *repetition.* Thus, we are justified in regarding the various degrees of mental activity from the simplest reflex manifestation to the highest mental act as a connected series of increasingly manifold and purposive actions proceeding from the same physico-chemical and physiological foundation.

57. **Feeling, Thinking, Acting.** For good reasons it is generally assumed that the organisms have not always been what they are now, but have "de-

veloped" from previous simpler forms. It is undecided whether originally there were one or several forms from which the present forms sprang, nor is it known how life first made its appearance on earth. So long as the various assumptions with regard to this question have not led to decisive, actually demonstrable differences in the results, a discussion of it is fruitless, and therefore unscientific. The usual word evolution is non-purposive in so far as it signifies the appearance of something already existing. Another conception is better according to which the influence of *changed* conditions of existence has yielded the most important factor of change.

The change that the organisms undergo is always in a definite direction. More and more complex and manifold forms are evolved, and the evolution of these forms is characterized by an ever greater specialization of the functions of life, so that every specially developed organ comes to perform but one function. It is true that by this means the organism is better fitted to perform those functions, but at the same time it grows more susceptible to injury, since its existence depends upon the proper simultaneous activity of many different organs. Such an evolution, therefore, can occur only when the general conditions of life have grown steadier, so that the danger of disturbance becomes less. We are accustomed to regard changes in this direction as higher developments, and the progressive sim-

plifications of the organization (as for example in parasites) as backward steps.

Since our opinion as to what constitutes a higher and a lower organism is doubtless arbitrary, let us ask whether it is not possible to find an *objective* standard by which to measure the relative perfection of the different organisms. The question must be answered in the affirmative when we take into consideration the following. Since the quantity of available free energy upon the earth is limited, the organism which transforms the energy at its disposal more completely and with the least loss into the forms of energy necessary for the function of life, must be regarded as the more perfect organism. In fact, we observe that with increasing complexity of the organisms there is for the most part also an increasing improvement in that direction, and we can therefore speak of some beings as more perfect than others. This view-point is especially significant in the evaluation of *human* progress, appearing, as it does, as the general standard of all civilization.

The perfection of the organism shows itself in relation to the outer world in the development of the *sense organs*. While a single-celled animal reacts almost exclusively to chemical, sometimes also to optical, stimuli, and receives these with the entire surface of its body, special parts of the body develop more and more toward perfection. These are the parts that respond with special ease to the ap-

propriate stimuli, that is, react to them with an increasingly smaller expenditure of energy. Then the points at which the stimuli are received are separated from those in which the reaction occurs, and the two are connected by *conducting paths,* the nerves, in which an energy process takes place. Our present knowledge of this process still leaves much to be desired. It is a process which moves with fairly great but by no means extraordinary rapidity (about ten to thirty meters per second) along the conducting paths. At the one end of this path it is caused by actions of various kinds, chiefly that of the specific energy, for which the sense organ is developed. At the other end it discharges specific effects. There is no doubt that here we have in each instance a case of energy transformation connected with a *discharge,* that is, with the action of other energies which lie at the ends ready for change. Hence there is no equivalence between the different kinds of energy, the discharging and the discharged, mostly not even a proportional relation, although both increase and decrease simultaneously.

What the form of the energy is that is propagated in the nerves is unknown. It can be either a special form which arises only under the conditions here present (just as, for example, a galvanic stream develops only under definite chemical and spacial conditions), or a special combination of known energies, as in sound and probably in light. Some day, it is likely, we shall have a more accurate knowl-

edge of the nerve process which will solve the question.

When such a process is caused by some energy impulse from without, it may produce various results. In the simplest case it discharges the corresponding reaction, just as the leaves of the sensitive plant close when they are touched. Or it may give rise to a series of processes following one another like the instinctive actions. Or, finally, it may bring about a series of inner processes which lead to an extreme differentiation of slight differences of this stimulus and to a corresponding graded reaction with the anticipation of success. We call this conscious thinking, willing, and acting.

Through the age-long effect of the blunder committed by Plato in making a fundamental distinction between mental life and physical life, we experience the utmost difficulty in habituating ourselves to the thought of the regular connection between the simplest physiological and the highest intellectual acts. Moreover, this contrast has been accentuated by the mechanical hypothesis. If we abandon the mechanical hypothesis and adhere to the summarization of experience free from all hypotheses, as represented in the science of energy, this contrast disappears. For even if we concede the impossibility of conceiving thought as *mechanical,* there is no difficulty in conceiving of it as *energetic,* especially since we know that mental work is connected with expenditure of energy and ex-

haustion just as physical work is. However, the elucidation of this subject lies almost entirely in the future since the idea just developed has but only begun to influence scientific work in this field. But judging from the results that have already been obtained we may hope for a speedy development.

58. Society. The external circumstance that as an organism multiplies the new being must come to life in the proximity of the older one, is in itself cause for the formation of closed groups confined to certain localities by animal organisms of the same species. But they become scattered if the advantage of their living together is not such as to outweigh the disadvantage of having a narrow field of competition for the means of sustenance. Thus we see different plants and animals behaving differently in this respect. While some species live in as great isolation as possible, others form communities, even if there is no mechanical tie to hold them together by a common integument.

Since the second case is true of man in a highly marked degree, his *social* characteristics and needs form a large and important part of his life. And since, further, the socialization of man makes continuous headway with increasing civilization—we need but think of the development of the former little groups and tribes into states and the present very active internationalization of the most important affairs of mankind, especially of the sci-

ences—the social problems also occupy an ever larger place in the organization of human life.

What distinguishes man most essentially from the other animals, even the most advanced, is his capacity for perfection, which in the lower animal can be paralleled at best by its capacity for *self-preservation.* While the organization of the animals within the short period of which we have any historical knowledge has to all appearances remained essentially unchanged, the world of mankind has changed in quite a remarkable way. This change consists in an increasing subjection of the external world to human purposes, and rests upon the increasing socialization of his capacities.

Memory and heredity (the latter being but an extension of memory to the offspring, which is to be conceived of as a part of the older organism) secures in the first place only the preservation of the stock and the renewed development of the new individual in the average type. If a specially favored individual succeeds in accomplishing greater achievements, he may in favorable circumstances transmit this capacity for higher attainments to his offspring. But such individuals gain an advantage in the struggle for existence only if the other sides of their activity do not suffer curtailment as a result. With the limited amount of energy at the individual's disposal every extraordinary accomplishment involves a corresponding *one-sidedness,* and as soon as a certain measure is slightly overstepped,

it will cause a reduction of the other functions which will render the individual less fit in the struggle for existence. But this is true only so long as an individual must live *by himself*. As soon as he forms part of a social organization which benefits by his particular activity, the organization compensates for the personal disadvantages by its collective activity, and a social community not only finds room for such special developments, but it even encourages and promotes them.

We have already seen that such manifestations occur within the organism itself. Higher functions, depending upon the higher susceptibility of the sense organs, can only be attained at the expense of the general functions by the organ in question. We observe this fact in all socially organized beings, like bees and ants, which display a high degree of specialization in the functions of the individual subordinate groups; the specialization often being carried so far that the individual groups can no longer subsist by themselves alone. It is only the organization as a whole that is capable of permanent existence.

While the evolution of such superior functions involves a corresponding differentiation, and therefore a *division* and *separation* of the superior functions within the social structure, the necessity for *communication* and for *mutual support* results in an *approximation* of the individuals and the groups. In every society, therefore, the centrifugal and the

centripetal forces work simultaneously in co-operation and in opposition to one another. While the extreme specialization on the one hand seems to make for the best individual functioning, on the other hand it renders the entire collective structure much more dependent, and therefore much more subject to injury, as is shown by the example of the queen bee, whose departure threatens the existence of the entire hive. Thus a medium degree of differentiation will as a general rule produce the most permanent social structure.

59. Language and Intercourse. The essential value of the social organization resides in the fact that the work of the individual, in so far as it is adapted to it, accrues to the benefit of the collective whole. For this it is absolutely essential that the members of the collectivity should be able to *have intercourse* with one another in order that every part of the general activity may be communicated to the others. This intercourse is obtained through language in the most general sense.

We have already learned that the essence of language consists in the co-ordination of concept to sign. The social application of language demands that the signs co-ordinated to the concepts in use should be the same for all the members of the social organization. Only in this way can the members make themselves mutually understood. But intelligible means of communication and division of labor impart to the social knowledge that is set

down in writing a kind of independent existence. Many centuries ago the possibility ceased for one person to store in his memory the entire stock of human knowledge. Nowadays we have men who are versed only in single parts of separate sciences, and the aggregate knowledge appears at first to be a unity existing only in thought. But because this knowledge is set down in signs which endure far beyond the life of the individual and at the appropriate moment can unfold its entire power even after a long period of inactivity, it has acquired an existence of a social character independent of the individual. For although it survives the individual, it cannot survive the death of human society.

As the socialization of all mankind advances to ever greater unities, the linguistic limitations sprung from former stages of evolution prove to be a hindrance. The mother tongue, of course, forms the first and most important entry for the individual to the common store of knowledge. But in view of the linguistic limitation of which I have just spoken the efforts in our day are carried on with renewed zeal to create a *universal auxiliary language* (p. 100) by means of which intercourse should be made possible beyond the language boundaries. There have already been gratifying results.*

* At the present time "Ido" is the best. It is a highly practicable artificial language, and its advocates have succeeded in organizing it to insure its normal development. An older and still rather widespread form called "Esperanto" has

60. Civilization. Everything which serves the social progress of mankind is appropriately called civilization or culture, and the objective characteristic of progress consists in improved methods for seizing and utilizing the raw energies of nature for human purposes. Thus it was a cultural act when a primitive man discovered that he could extend the radius of his muscle energy by taking a pole in his hand, and it was another cultural act when a primitive man discovered that by throwing a stone he could send his muscle energy a distance of many meters to the desired point. The effect of the knife, the spear, the arrow, and of all the other primitive implements can be called in each case a purposive transformation of energy. And at the other end of the scale of civilization the most abstract scientific discovery, by reason of its generalization and simplification, signifies a correspondng economy of energy for all the coming generations that may have anything to do with the matter. Thus, in fact, the concept of progress as here defined embraces the entire sweep of human endeavor for perfection, or the entire field of culture, and at the same time it shows the great scientific value of the concept of energy.

If we consider further that, according to the second fundamental principle, the free energy accessible to us can only decrease, but not increase, while the number of men whose existence depends directly

failed to organize itself so as to insure its development and it must inevitably die out.

on the consumption of a due amount of free energy is constantly on the increase, then we at once see the objective necessity of the development of civilization in that sense. His foresight puts man in a position to act culturally. But if we examine our present social order from this point of view, we realize with horror how barbarous it still is. Not only do murder and war destroy cultural values without substituting others in their place, not only do the countless conflicts which take place between the different nations and political organizations act anticulturally, but so do also the conflicts between the various social classes of one nation, for they destroy quantities of free energy which are thus withdrawn from the total of real cultural values. At present mankind is in a state of development in which progress depends much less upon the leadership of a few distinguished individuals than upon the collective labor of all workers. Proof of this is that it is coming to be more and more the fact that the great scientific discoveries are made simultaneously by a number of independent investigators —an indication that society creates in several places the individual conditions requisite for such discoveries. Thus we are living at a time when men are gradually approximating one another very closely in their natures, and when the social organization therefore demands and strives for as thorough an equalization as possible in the conditions of existence of all men.

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